

## Problem Solutions for Chapter 2

2-1. 
$$\mathbf{E} = 100 \cos (2\pi 10^8 t + 30^\circ) \mathbf{e}_x + 20 \cos (2\pi 10^8 t - 50^\circ) \mathbf{e}_y$$
$$+ 40 \cos (2\pi 10^8 t + 210^\circ) \mathbf{e}_z$$

2-2. The general form is:

$y = (\text{amplitude}) \cos(\omega t - kz) = A \cos [2\pi(vt - z/\lambda)]$ . Therefore

(a) amplitude = 8  $\mu\text{m}$

(b) wavelength:  $1/\lambda = 0.8 \mu\text{m}^{-1}$  so that  $\lambda = 1.25 \mu\text{m}$

(c)  $\omega = 2\pi\nu = 2\pi(2) = 4\pi$

(d) At  $t = 0$  and  $z = 4 \mu\text{m}$  we have

$$y = 8 \cos [2\pi(-0.8 \mu\text{m}^{-1})(4 \mu\text{m})]$$
$$= 8 \cos [2\pi(-3.2)] = 2.472$$

2-3. For  $E$  in electron volts and  $\lambda$  in  $\mu\text{m}$  we have  $E = \frac{1.240}{\lambda}$

(a) At 0.82  $\mu\text{m}$ ,  $E = 1.240/0.82 = 1.512 \text{ eV}$

At 1.32  $\mu\text{m}$ ,  $E = 1.240/1.32 = 0.939 \text{ eV}$

At 1.55  $\mu\text{m}$ ,  $E = 1.240/1.55 = 0.800 \text{ eV}$

(b) At 0.82  $\mu\text{m}$ ,  $k = 2\pi/\lambda = 7.662 \mu\text{m}^{-1}$

At 1.32  $\mu\text{m}$ ,  $k = 2\pi/\lambda = 4.760 \mu\text{m}^{-1}$

At 1.55  $\mu\text{m}$ ,  $k = 2\pi/\lambda = 4.054 \mu\text{m}^{-1}$

2-4.  $x_1 = a_1 \cos (\omega t - \delta_1)$  and  $x_2 = a_2 \cos (\omega t - \delta_2)$

Adding  $x_1$  and  $x_2$  yields

$$x_1 + x_2 = a_1 [\cos \omega t \cos \delta_1 + \sin \omega t \sin \delta_1]$$
$$+ a_2 [\cos \omega t \cos \delta_2 + \sin \omega t \sin \delta_2]$$
$$= [a_1 \cos \delta_1 + a_2 \cos \delta_2] \cos \omega t + [a_1 \sin \delta_1 + a_2 \sin \delta_2] \sin \omega t$$

Since the  $a$ 's and the  $\delta$ 's are constants, we can set

$$a_1 \cos \delta_1 + a_2 \cos \delta_2 = A \cos \phi \quad (1)$$

$$a_1 \sin \delta_1 + a_2 \sin \delta_2 = A \sin \phi \quad (2)$$

provided that constant values of A and  $\phi$  exist which satisfy these equations. To verify this, first square both sides and add:

$$A^2 (\sin^2 \phi + \cos^2 \phi) = a_1^2 (\sin^2 \delta_1 + \cos^2 \delta_1) + a_2^2 (\sin^2 \delta_2 + \cos^2 \delta_2) + 2a_1 a_2 (\sin \delta_1 \sin \delta_2 + \cos \delta_1 \cos \delta_2)$$

or

$$A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos (\delta_1 - \delta_2)$$

Dividing (2) by (1) gives

$$\tan \phi = \frac{a_1 \sin \delta_1 + a_2 \sin \delta_2}{a_1 \cos \delta_1 + a_2 \cos \delta_2}$$

Thus we can write

$$x = x_1 + x_2 = A \cos \phi \cos \omega t + A \sin \phi \sin \omega t = A \cos(\omega t - \phi)$$

2-5. First expand Eq. (2-3) as

$$\frac{E_y}{E_{0y}} = \cos (\omega t - kz) \cos \delta - \sin (\omega t - kz) \sin \delta \quad (2.5-1)$$

Subtract from this the expression

$$\frac{E_x}{E_{0x}} \cos \delta = \cos (\omega t - kz) \cos \delta$$

to yield

$$\frac{E_y}{E_{0y}} - \frac{E_x}{E_{0x}} \cos \delta = - \sin (\omega t - kz) \sin \delta \quad (2.5-2)$$

Using the relation  $\cos^2 \alpha + \sin^2 \alpha = 1$ , we use Eq. (2-2) to write

$$\sin^2 (\omega t - kz) = [1 - \cos^2 (\omega t - kz)] = \left[ 1 - \left( \frac{E_x}{E_{0x}} \right)^2 \right] \quad (2.5-3)$$

Squaring both sides of Eq. (2.5-2) and substituting it into Eq. (2.5-3) yields

$$\left[ \frac{E_y}{E_{0y}} - \frac{E_x}{E_{0x}} \cos \delta \right]^2 = \left[ 1 - \left( \frac{E_x}{E_{0x}} \right)^2 \right] \sin^2 \delta$$

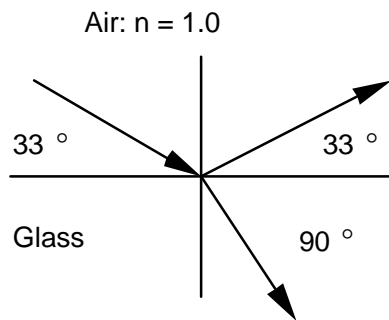
Expanding the left-hand side and rearranging terms yields

$$\left( \frac{E_x}{E_{0x}} \right)^2 + \left( \frac{E_y}{E_{0y}} \right)^2 - 2 \left( \frac{E_x}{E_{0x}} \right) \left( \frac{E_y}{E_{0y}} \right) \cos \delta = \sin^2 \delta$$

2-6. Plot of Eq. (2-7).

2-7. Linearly polarized wave.

2-8.



(a) Apply Snell's law

$$n_1 \cos \theta_1 = n_2 \cos \theta_2$$

where  $n_1 = 1$ ,  $\theta_1 = 33^\circ$ , and  $\theta_2 = 90^\circ - 33^\circ = 57^\circ$

$$\therefore n_2 = \frac{\cos 33^\circ}{\cos 57^\circ} = 1.540$$

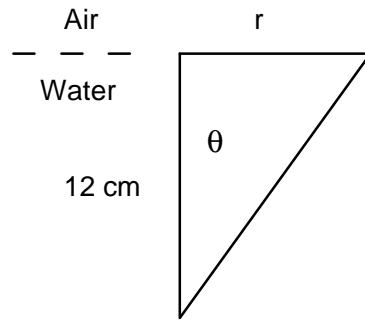
(b) The critical angle is found from

$$n_{\text{glass}} \sin \phi_{\text{glass}} = n_{\text{air}} \sin \phi_{\text{air}}$$

with  $\phi_{\text{air}} = 90^\circ$  and  $n_{\text{air}} = 1.0$

$$\therefore \phi_{\text{critical}} = \arcsin \frac{1}{n_{\text{glass}}} = \arcsin \frac{1}{1.540} = 40.5^\circ$$

2-9

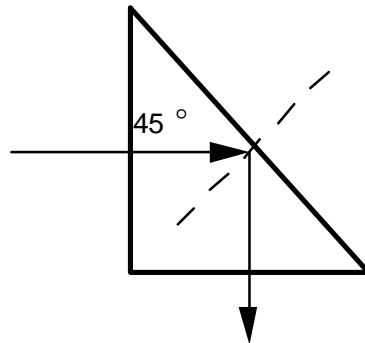


Find  $\theta_c$  from Snell's law  $n_1 \sin \theta_1 = n_2 \sin \theta_c = 1$

When  $n_2 = 1.33$ , then  $\theta_c = 48.75^\circ$

Find  $r$  from  $\tan \theta_c = \frac{r}{12 \text{ cm}}$ , which yields  $r = 13.7 \text{ cm}$ .

2-10.



Using Snell's law  $n_{\text{glass}} \sin \theta_c = n_{\text{alcohol}} \sin 90^\circ$

where  $\theta_c = 45^\circ$  we have

$$n_{\text{glass}} = \frac{1.45}{\sin 45^\circ} = 2.05$$

2-11. (a) Use either  $NA = (n_1^2 - n_2^2)^{1/2} = \underline{0.242}$

or

$$NA \approx n_1 \sqrt{2\Delta} = n_1 \sqrt{\frac{2(n_1^2 - n_2^2)}{n_1^2}} = \underline{0.243}$$

$$(b) \theta_{0,\max} = \arcsin (NA/n) = \arcsin \left( \frac{0.242}{1.0} \right) = 14^\circ$$

$$2-13. \quad NA = (n_1^2 - n_2^2)^{1/2} = [n_1^2 - n_1^2(1 - \Delta)^2]^{1/2}$$

$$= n_1 (2\Delta - \Delta^2)^{1/2}$$

$$\text{Since } \Delta \ll 1, \Delta^2 \ll \Delta; \therefore NA \approx n_1 \sqrt{2\Delta}$$

2-14. (a) Solve Eq. (2-34a) for  $jH_\phi$ :

$$jH_\phi = j \frac{\epsilon\omega}{\beta} E_r - \frac{1}{\beta r} \frac{\partial H_z}{\partial \phi} \quad \text{Substituting into Eq. (2-33b) we have}$$

$$j \beta E_r + \frac{\partial E_z}{\partial r} = \omega\mu \left[ j \frac{\epsilon\omega}{\beta} E_r - \frac{1}{\beta r} \frac{\partial H_z}{\partial \phi} \right]$$

Solve for  $E_r$  and let  $q^2 = \omega^2\epsilon\mu - \beta^2$  to obtain Eq. (2-35a).

(b) Solve Eq. (2-34b) for  $jH_r$ :

$$jH_r = -j \frac{\epsilon\omega}{\beta} E_\phi - \frac{1}{\beta} \frac{\partial H_z}{\partial r} \quad \text{Substituting into Eq. (2-33a) we have}$$

$$j \beta E_\phi + \frac{1}{r} \frac{\partial E_z}{\partial \phi} = -\omega\mu \left[ -j \frac{\epsilon\omega}{\beta} E_\phi - \frac{1}{\beta} \frac{\partial H_z}{\partial r} \right]$$

Solve for  $E_\phi$  and let  $q^2 = \omega^2\epsilon\mu - \beta^2$  to obtain Eq. (2-35b).

(c) Solve Eq. (2-34a) for  $jE_r$ :

$$jE_r = \frac{1}{\epsilon\omega} \frac{1}{r} \left( \frac{\partial H_z}{\partial \phi} + jr\beta H_\phi \right) \quad \text{Substituting into Eq. (2-33b) we have}$$

$$\frac{\beta}{\epsilon\omega} \frac{1}{r} \left( \frac{\partial H_z}{\partial \phi} + jr\beta H_\phi \right) + \frac{\partial E_z}{\partial r} = j\omega\mu H_\phi$$

Solve for  $H_\phi$  and let  $q^2 = \omega^2\epsilon\mu - \beta^2$  to obtain Eq. (2-35d).

(d) Solve Eq. (2-34b) for  $jE_\phi$

$$jE_\phi = - \frac{1}{\epsilon\omega} \left( j\beta H_r + \frac{\partial H_z}{\partial r} \right) \quad \text{Substituting into Eq. (2-33a) we have}$$

$$\frac{1}{r} \frac{\partial E_z}{\partial \phi} - \frac{\beta}{\epsilon\omega} \left( j\beta H_r + \frac{\partial H_z}{\partial r} \right) = -j\omega\mu H_r$$

Solve for  $H_r$  to obtain Eq. (2-35c).

(e) Substitute Eqs. (2-35c) and (2-35d) into Eq. (2-34c)

$$- \frac{j}{q^2} \frac{1}{r} \left[ \frac{\partial}{\partial r} \left( \beta \frac{\partial H_z}{\partial \phi} + \epsilon\omega r \frac{\partial E_z}{\partial r} \right) - \frac{\partial}{\partial \phi} \left( \beta \frac{\partial H_z}{\partial r} - \frac{\epsilon\omega}{r} \frac{\partial E_z}{\partial \phi} \right) \right] = j\epsilon\omega E_z$$

Upon differentiating and multiplying by  $jq^2/\epsilon\omega$  we obtain Eq. (2-36).

(f) Substitute Eqs. (2-35a) and (2-35b) into Eq. (2-33c)

$$- \frac{j}{q^2} \frac{1}{r} \left[ \frac{\partial}{\partial r} \left( \beta \frac{\partial E_z}{\partial \phi} - \mu\omega r \frac{\partial H_z}{\partial r} \right) - \frac{\partial}{\partial \phi} \left( \beta \frac{\partial E_z}{\partial r} + \frac{\mu\omega}{r} \frac{\partial H_z}{\partial \phi} \right) \right] = -j\mu\omega H_z$$

Upon differentiating and multiplying by  $jq^2/\epsilon\omega$  we obtain Eq. (2-37).

2-15. For  $v = 0$ , from Eqs. (2-42) and (2-43) we have

$$E_z = AJ_0(ur) e^{j(\alpha x - \beta z)} \quad \text{and} \quad H_z = BJ_0(ur) e^{j(\alpha x - \beta z)}$$

We want to find the coefficients A and B. From Eqs. (2-47) and (2-51), respectively, we have

$$C = \frac{J_v(ua)}{K_v(wa)} A \quad \text{and} \quad D = \frac{J_v(ua)}{K_v(wa)} B$$

Substitute these into Eq. (2-50) to find B in terms of A:

$$A \left( \frac{j\beta v}{a} \right) \left( \frac{1}{u^2} + \frac{1}{w^2} \right) = B \omega \mu \left[ \frac{J'_v(ua)}{u J_v(ua)} + \frac{K'_v(wa)}{w K_v(wa)} \right]$$

For  $v = 0$ , the right-hand side must be zero. Also for  $v = 0$ , either Eq. (2-55a) or (2-56a) holds. Suppose Eq. (2-56a) holds, so that the term in square brackets on the right-hand side in the above equation is not zero. Then we must have that  $B = 0$ , which from Eq. (2-43) means that  $H_z = 0$ . Thus Eq. (2-56) corresponds to  $TM_{0m}$  modes.

For the other case, substitute Eqs. (2-47) and (2-51) into Eq. (2-52):

$$0 = \frac{1}{u^2} \left[ B \frac{j\beta v}{a} J'_v(ua) + A \omega \epsilon_1 u J'_v(ua) \right] \\ + \frac{1}{w^2} \left[ B \frac{j\beta v}{a} J'_v(wa) + A \omega \epsilon_2 w \frac{K'_v(wa) J_v(ua)}{K_v(wa)} \right]$$

With  $k_1^2 = \omega^2 \mu \epsilon_1$  and  $k_2^2 = \omega^2 \mu \epsilon_2$  rewrite this as

$$Bv = \frac{ja}{\beta \omega \mu} \left[ \frac{1}{\frac{1}{u^2} + \frac{1}{w^2}} \right] [k_1^2 J_v + k_2^2 K_v] A$$

where  $J_v$  and  $K_v$  are defined in Eq. (2-54). If for  $v = 0$  the term in square brackets on the right-hand side is non-zero, that is, if Eq. (2-56a) does not hold, then we must have that  $A = 0$ , which from Eq. (2-42) means that  $E_z = 0$ . Thus Eq. (2-55) corresponds to  $TE_{0m}$  modes.

2-16. From Eq. (2-23) we have

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2} = \frac{1}{2} \left( 1 - \frac{n_2^2}{n_1^2} \right)$$

$$\Delta \ll 1 \quad \text{implies } n_1 \approx n_2$$

Thus using Eq. (2-46), which states that  $n_2 k = k_2 \leq \beta \leq k_1 = n_1 k$ , we have

$$n_2^2 k^2 = k_2^2 \approx n_1^2 k^2 = k_1^2 \approx \beta^2$$

2-17.

2-18. (a) From Eqs. (2-59) and (2-61) we have

$$M \approx \frac{2\pi^2 a^2}{\lambda^2} (n_1^2 - n_2^2) = \frac{2\pi^2 a^2}{\lambda^2} (NA)^2$$

$$a = \left( \frac{M}{2\pi} \right)^{1/2} \frac{\lambda}{NA} = \left( \frac{1000}{2} \right)^{1/2} \frac{0.85 \mu\text{m}}{0.2\pi} = 30.25 \mu\text{m}$$

Therefore,  $D = 2a = 60.5 \mu\text{m}$

$$(b) M = \frac{2\pi^2 (30.25 \mu\text{m})^2}{(1.32 \mu\text{m})^2} (0.2)^2 = 414$$

(c) At 1550 nm,  $M = 300$

2-19. From Eq. (2-58),



$$V = \frac{2\pi (25 \mu\text{m})}{0.82 \mu\text{m}} \left[ (1.48)^2 - (1.46)^2 \right]^{1/2} = 46.5$$

Using Eq. (2-61)  $M \approx V^2/2 = 1081$  at 820 nm.

Similarly,  $M = 417$  at 1320 nm and  $M = 303$  at 1550 nm. From Eq. (2-72)

$$\left( \frac{P_{\text{clad}}}{P} \right)_{\text{total}} \approx \frac{4}{3} M^{-1/2} = \frac{4 \times 100\%}{3\sqrt{1080}} = 4.1\%$$

at 820 nm. Similarly,  $(P_{\text{clad}}/P)_{\text{total}} = 6.6\%$  at 1320 nm and  $7.8\%$  at 1550 nm.

2-20 (a) At 1320 nm we have from Eqs. (2-23) and (2-57) that  $V = 25$  and  $M = 312$ .

(b) From Eq. (2-72) the power flow in the cladding is 7.5%.

2-21. (a) For single-mode operation, we need  $V \leq 2.40$ .

Solving Eq. (2-58) for the core radius  $a$

$$a = \frac{V\lambda}{2\pi} (n_1^2 - n_2^2)^{-1/2} = \frac{2.40(1.32\mu\text{m})}{2\pi[(1.480)^2 - (1.478)^2]^{1/2}} = 6.55 \mu\text{m}$$

(b) From Eq. (2-23)

$$\text{NA} = (n_1^2 - n_2^2)^{1/2} = [(1.480)^2 - (1.478)^2]^{1/2} = 0.077$$

(c) From Eq. (2-23),  $\text{NA} = n \sin \theta_{0,\text{max}}$ . When  $n = 1.0$  then

$$\theta_{0,\text{max}} = \arcsin \left( \frac{\text{NA}}{n} \right) = \arcsin \left( \frac{0.077}{1.0} \right) = 4.4^\circ$$

$$2-22. \quad n_2 = \sqrt{n_1^2 - \text{NA}^2} = \sqrt{(1.458)^2 - (0.3)^2} = 1.427$$

$$a = \frac{\lambda V}{2\pi \text{NA}} = \frac{(1.30)(75)}{2\pi(0.3)} = 52 \mu\text{m}$$

2-23. For small values of  $\Delta$  we can write  $V \approx \frac{2\pi a}{\lambda} n_1 \sqrt{2\Delta}$

For  $a = 5 \mu\text{m}$  we have  $\Delta \approx 0.002$ , so that at  $0.82 \mu\text{m}$

$$V \approx \frac{2\pi (5 \mu\text{m})}{0.82 \mu\text{m}} 1.45 \sqrt{2(0.002)} = 3.514$$

Thus the fiber is no longer single-mode. From Figs. 2-18 and 2-19 we see that the  $\text{LP}_{01}$  and the  $\text{LP}_{11}$  modes exist in the fiber at  $0.82 \mu\text{m}$ .

2-24.

2-25. From Eq. (2-77)  $L_p = \frac{2\pi}{\beta} = \frac{\lambda}{n_y - n_x}$

$$\text{For } L_p = 10 \text{ cm} \quad n_y - n_x = \frac{1.3 \times 10^{-6} \text{ m}}{10^{-1} \text{ m}} = 1.3 \times 10^{-5}$$

$$\text{For } L_p = 2 \text{ m} \quad n_y - n_x = \frac{1.3 \times 10^{-6} \text{ m}}{2 \text{ m}} = 6.5 \times 10^{-7}$$

Thus

$$6.5 \times 10^{-7} \leq n_y - n_x \leq 1.3 \times 10^{-5}$$

2-26. We want to plot  $n(r)$  from  $n_2$  to  $n_1$ . From Eq. (2-78)

$$n(r) = n_1 \left[ 1 - 2\Delta(r/a)^\alpha \right]^{1/2} = 1.48 \left[ 1 - 0.02(r/25)^\alpha \right]^{1/2}$$

$$n_2 \text{ is found from Eq. (2-79): } n_2 = n_1(1 - \Delta) = 1.465$$

2-27. From Eq. (2-81)

$$M = \frac{\alpha}{\alpha + 2} a^2 k^2 n_1^2 \Delta = \frac{\alpha}{\alpha + 2} \left( \frac{2\pi a n_1}{\lambda} \right)^2 \Delta$$

where

$$\Delta = \frac{n_1 - n_2}{n_1} = 0.0135$$

At  $\lambda = 820$  nm,  $M = 543$  and at  $\lambda = 1300$  nm,  $M = 216$ .

For a step index fiber we can use Eq. (2-61)

$$M_{\text{step}} \approx \frac{V^2}{2} = \frac{1}{2} \left( \frac{2\pi a}{\lambda} \right)^2 (n_1^2 - n_2^2)$$

At  $\lambda = 820$  nm,  $M_{\text{step}} = 1078$  and at  $\lambda = 1300$  nm,  $M_{\text{step}} = 429$ .

Alternatively, we can let  $\alpha = \infty$  in Eq. (2-81):

$$M_{\text{step}} = \left( \frac{2\pi a n_1}{\lambda} \right)^2 \Delta = \begin{cases} 1086 & \text{at } 820 \text{ nm} \\ 432 & \text{at } 1300 \text{ nm} \end{cases}$$

2-28. Using Eq. (2-23) we have

$$(a) \text{ NA} = (n_1^2 - n_2^2)^{1/2} = [(1.60)^2 - (1.49)^2]^{1/2} = 0.58$$

$$(b) \text{ NA} = [(1.458)^2 - (1.405)^2]^{1/2} = 0.39$$

2-29. (a) From the Principle of the Conservation of Mass, the volume of a preform rod section of length  $L_{\text{preform}}$  and cross-sectional area  $A$  must equal the volume of the fiber drawn from this section. The preform section of length  $L_{\text{preform}}$  is drawn into a fiber of length  $L_{\text{fiber}}$  in a time  $t$ . If  $S$  is the preform feed speed, then  $L_{\text{preform}} = St$ . Similarly, if  $s$  is the fiber drawing speed, then  $L_{\text{fiber}} = st$ . Thus, if  $D$  and  $d$  are the preform and fiber diameters, respectively, then

$$\text{Preform volume} = L_{\text{preform}}(D/2)^2 = St (D/2)^2$$

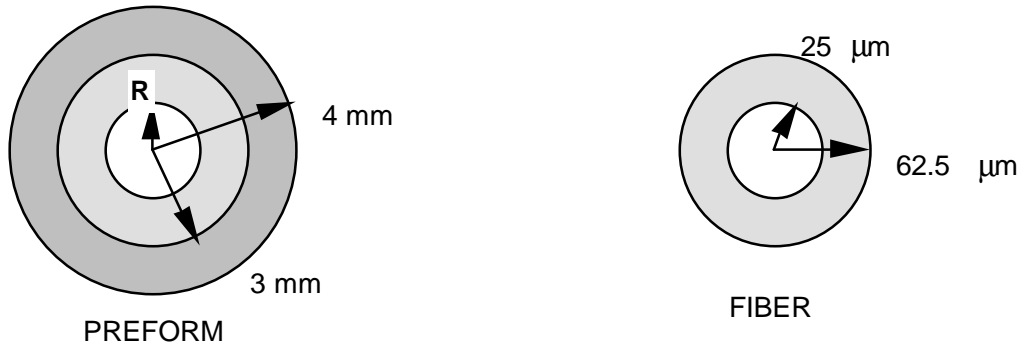
$$\text{and} \quad \text{Fiber volume} = L_{\text{fiber}} (d/2)^2 = st (d/2)^2$$

Equating these yields

$$St \left( \frac{D}{2} \right)^2 = st \left( \frac{d}{2} \right)^2 \quad \text{or} \quad s = S \left( \frac{D}{d} \right)^2$$

$$(b) \quad S = s \left( \frac{d}{D} \right)^2 = 1.2 \text{ m/s} \left( \frac{0.125 \text{ mm}}{9 \text{ mm}} \right)^2 = 1.39 \text{ cm/min}$$

2-30. Consider the following geometries of the preform and its corresponding fiber:



We want to find the thickness of the deposited layer ( $3 \text{ mm} - R$ ). This can be done by comparing the ratios of the preform core-to-cladding cross-sectional areas and the fiber core-to-cladding cross-sectional areas:

$$\frac{A_{\text{preform core}}}{A_{\text{preform clad}}} = \frac{A_{\text{fiber core}}}{A_{\text{fiber clad}}}$$

or

$$\frac{\pi(3^2 - R^2)}{\pi(4^2 - 3^2)} = \frac{\pi(25)^2}{\pi[(62.5)^2 - (25)^2]}$$

from which we have

$$R = \left[ 9 - \frac{7(25)^2}{(62.5)^2 - (25)^2} \right]^{1/2} = 2.77 \text{ mm}$$

Thus, thickness =  $3 \text{ mm} - 2.77 \text{ mm} = 0.23 \text{ mm}$ .

2-31. (a) The volume of a 1-km-long 50-μm diameter fiber core is

$$V = \pi r^2 L = \pi (2.5 \times 10^{-3} \text{ cm})^2 (10^5 \text{ cm}) = 1.96 \text{ cm}^3$$

The mass  $M$  equals the density  $\rho$  times the volume  $V$ :

$$M = \rho V = (2.6 \text{ gm/cm}^3)(1.96 \text{ cm}^3) = 5.1 \text{ gm}$$

(b) If R is the deposition rate, then the deposition time t is

$$t = \frac{M}{R} = \frac{5.1 \text{ gm}}{0.5 \text{ gm/min}} = 10.2 \text{ min}$$

2-32. Solving Eq. (2-82) for  $\chi$  yields

$$\chi = \left( \frac{K}{Y\sigma} \right)^2 \quad \text{where } Y = \sqrt{\pi} \quad \text{for surface flaws.}$$

Thus

$$\chi = \frac{(20 \text{ N/mm}^{3/2})^2}{(70 \text{ MN/m}^2)^2 \pi} = 2.60 \times 10^{-4} \text{ mm} = 0.26 \text{ }\mu\text{m}$$

2-33. (a) To find the time to failure, we substitute Eq. (2-82) into Eq. (2-86) and integrate (assuming that  $\sigma$  is independent of time):

$$\int_{\chi_i}^{\chi_f} \chi^{-b/2} d\chi = AY^b \sigma^b \int_0^t dt$$

which yields

$$\frac{1}{1 - \frac{b}{2}} \left[ \chi_f^{1-b/2} - \chi_i^{1-b/2} \right] = AY^b \sigma^b t$$

or

$$t = \frac{2}{(b-2)A(Y\sigma)^b} \left[ \chi_i^{(2-b)/2} - \chi_f^{(2-b)/2} \right]$$

(b) Rewriting the above expression in terms of K instead of  $\chi$  yields

$$t = \frac{2}{(b-2)A(Y\sigma)^b} \left[ \left( \frac{K_i}{Y\sigma} \right)^{2-b} - \left( \frac{K_f}{Y\sigma} \right)^{2-b} \right]$$

$$\approx \frac{2K_i^{2-b}}{(b-2)A(Y\sigma)^b} \quad \text{if } K_i^{b-2} \ll K_f^{b-2} \quad \text{or} \quad K_i^{2-b} \gg K_f^{2-b}$$

2-34. Substituting Eq. (2-82) into Eq. (2-86) gives

$$\frac{d\chi}{dt} = AK^b = AY^b \chi^{b/2} \sigma^b$$

Integrating this from  $\chi_i$  to  $\chi_p$  where

$$\chi_i = \left( \frac{K}{Y\sigma_i} \right)^2 \quad \text{and} \quad \chi_p = \left( \frac{K}{Y\sigma_p} \right)^2$$

are the initial crack depth and the crack depth after proof testing, respectively, yields

$$\int_{\chi_i}^{\chi_p} \chi^{-b/2} d\chi = AY^b \int_0^{t_p} \sigma^b dt$$

or

$$\frac{1}{1 - \frac{b}{2}} \left[ \chi_p^{1-b/2} - \chi_i^{1-b/2} \right] = AY^b \sigma_p^b t_p$$

for a constant stress  $\sigma_p$ . Substituting for  $\chi_i$  and  $\chi_p$  gives

$$\left( \frac{2}{b-2} \right) \left( \frac{K}{Y} \right)^{2-b} \left[ \sigma_i^{b-2} - \sigma_p^{b-2} \right] = AY^b \sigma_p^b t_p$$

or

$$\left( \frac{2}{b-2} \right) \left( \frac{K}{Y} \right)^{2-b} \frac{1}{AY^b} \left[ \sigma_i^{b-2} - \sigma_p^{b-2} \right] = B \left[ \sigma_i^{b-2} - \sigma_p^{b-2} \right] = \sigma_p^b t_p$$

which is Eq. (2-87).

When a static stress  $\sigma_s$  is applied after proof testing, the time to failure is found from Eq. (2-86):

$$\int_{\chi_p}^{\chi_s} \chi^{-b/2} d\chi = AY^b \sigma_s^b \int_0^{t_s} dt$$

where  $\chi_s$  is the crack depth at the fiber failure point. Integrating (as above) we get Eq. (2-89):

$$B \left[ \sigma_p^{b-2} - \sigma_s^{b-2} \right] = \sigma_s^b t_s$$

Adding Eqs. (2-87) and (2-89) yields Eq. (2-90).

2-35. (a) Substituting  $N_s$  as given by Eq. (2-92) and  $N_p$  as given by Eq. (2-93) into Eq. (2-94) yields

$$F = 1 - \exp \left\{ -\frac{L}{L_0} \left\{ \frac{\left[ (\sigma_p^b t_p + \sigma_s^b t_s) / B + \sigma_s^{b-2} \right]^{\frac{m}{b-2}}}{\sigma_0^m} - \frac{(\sigma_p^b t_p / B + \sigma_p^{b-2})^{\frac{m}{b-2}}}{\sigma_0^m} \right\} \right\}$$

$$= 1 - \exp \left\{ -\frac{L}{L_0 \sigma_0^m} \left[ \sigma_p^b t_p / B + \sigma_p^{b-2} \right]^{\frac{m}{b-2}} \left\{ \frac{\left[ \left( \frac{\sigma_p^b t_p + \sigma_s^b t_s}{B} \right) + \sigma_s^{b-2} \right]^{\frac{m}{b-2}}}{\sigma_p^b t_p / B + \sigma_p^{b-2}} \right\} - 1 \right\}$$

$$= 1 - \exp \left\{ -LN_p \left\{ \frac{\left[ 1 + \frac{\sigma_s^b t_s}{\sigma_p^b t_p} + \left( \frac{\sigma_s}{\sigma_p} \right)^b \frac{B}{\sigma_s^2 t_p} \right]^{\frac{m}{b-2}}}{1 + \frac{B}{\sigma_p^2 t_p}} - 1 \right\} \right\}$$

$$\approx 1 - \exp \left\{ -LN_p \left\{ \left[ \left( 1 + \frac{\sigma_s^b t_s}{\sigma_p^b t_p} \right) \frac{1}{1 + \frac{B}{\sigma_p^2 t_p}} \right]^{\frac{m}{b-2}} - 1 \right\} \right\}$$

(b) For the term given by Eq. (2-96) we have

$$\left( \frac{\sigma_s}{\sigma_p} \right)^b \frac{B}{\sigma_s^2 t_p} = (0.3)^{15} \frac{0.5 \text{ (MN/m}^2\text{)}^2 \text{ s}}{\left[ 0.3 (350 \text{ MN/m}^2) \right]^2 10 \text{ s}} = 6.5 \times 10^{-14}$$

Thus this term can be neglected.

2-36. The failure probability is given by Eq. (2-85). For equal failure probabilities of the two fiber samples,  $F_1 = F_2$ , or

$$1 - \exp\left[-\left(\frac{\sigma_{1c}}{\sigma_0}\right)^m \frac{L_1}{L_0}\right] = 1 - \exp\left[-\left(\frac{\sigma_{2c}}{\sigma_0}\right)^m \frac{L_2}{L_0}\right]$$

which implies that

$$\left(\frac{\sigma_{1c}}{\sigma_0}\right)^m \frac{L_1}{L_0} = \left(\frac{\sigma_{2c}}{\sigma_0}\right)^m \frac{L_2}{L_0}$$

or

$$\frac{\sigma_{1c}}{\sigma_{2c}} = \left(\frac{L_2}{L_1}\right)^{1/m}$$

If  $L_1 = 20$  m, then  $\sigma_{1c} = 4.8$  GN/m<sup>2</sup>

If  $L_2 = 1$  km, then  $\sigma_{2c} = 3.9$  GN/m<sup>2</sup>

Thus

$$\left(\frac{4.8}{3.9}\right)^m = \frac{1000}{20} = 50$$

gives

$$m = \frac{\log 50}{\log(4.8/3.9)} = 18.8$$



### Problem Solutions for Chapter 3

3-1.

$$\alpha(\text{dB/km}) = \frac{10}{z} \log \left[ \frac{P(0)}{P(z)} \right] = \frac{10}{z} \log \left( e^{\alpha_p z} \right)$$
$$= 10\alpha_p \log e = 4.343 \alpha_p (1/\text{km})$$

3-2. Since the attenuations are given in dB/km, first find the power levels in dBm for 100  $\mu\text{W}$  and 150  $\mu\text{W}$ . These are, respectively,

$$P(100 \mu\text{W}) = 10 \log (100 \mu\text{W}/1.0 \text{ mW}) = 10 \log (0.10) = -10.0 \text{ dBm}$$

$$P(150 \mu\text{W}) = 10 \log (150 \mu\text{W}/1.0 \text{ mW}) = 10 \log (0.15) = -8.24 \text{ dBm}$$

(a) At 8 km we have the following power levels:

$$P_{1300}(8 \text{ km}) = -8.2 \text{ dBm} - (0.6 \text{ dB/km})(8 \text{ km}) = -13.0 \text{ dBm} = 50 \mu\text{W}$$

$$P_{1550}(8 \text{ km}) = -10.0 \text{ dBm} - (0.3 \text{ dB/km})(8 \text{ km}) = -12.4 \text{ dBm} = 57.5 \mu\text{W}$$

(b) At 20 km we have the following power levels:

$$P_{1300}(20 \text{ km}) = -8.2 \text{ dBm} - (0.6 \text{ dB/km})(20 \text{ km}) = -20.2 \text{ dBm} = 9.55 \mu\text{W}$$

$$P_{1550}(20 \text{ km}) = -10.0 \text{ dBm} - (0.3 \text{ dB/km})(20 \text{ km}) = -16.0 \text{ dBm} = 25.1 \mu\text{W}$$

3-3. From Eq. (3-1c) with  $P_{\text{out}} = 0.45 P_{\text{in}}$

$$\alpha = (10/3.5 \text{ km}) \log (1/0.45) = 1.0 \text{ dB/km}$$

3-4. (a)  $P_{\text{in}} = P_{\text{out}} 10^{\alpha L/10} = (0.3 \mu\text{W}) 10^{1.5(12)/10} = 18.9 \mu\text{W}$

$$(b) \quad P_{\text{in}} = P_{\text{out}} 10^{\alpha L/10} = (0.3 \mu\text{W}) 10^{2.5(12)/10} = 300 \mu\text{W}$$

3-5. With  $\lambda$  in Eqs. (3-2b) and (3-3) given in  $\mu\text{m}$ , we have the following representative points for  $\alpha_{\text{UV}}$  and  $\alpha_{\text{IR}}$ :

$\lambda$ ( $\mu\text{m}$ )	$\alpha_{\text{uv}}$	$\alpha_{\text{IR}}$
0.5	20.3	--
0.7	1.44	--
0.9	0.33	--
1.2	0.09	$2.2 \times 10^{-6}$
1.5	0.04	0.0072
2.0	0.02	23.2
3.0	0.009	$7.5 \times 10^4$

3-6. From Eq. (3-4a) we have

$$\begin{aligned}
 \alpha_{\text{scat}} &= \frac{8\pi^3}{3\lambda^4} (n^2 - 1)^2 k_B T_f \beta_T \\
 &= \frac{8\pi^3}{3(0.63 \mu\text{m})^4} [(1.46)^2 - 1]^2 (1.38 \times 10^{-16} \text{ dyne-cm/K})(1400 \text{ K}) \\
 &\quad \times (6.8 \times 10^{-12} \text{ cm}^2/\text{dyne}) \\
 &= 0.883 \text{ km}^{-1}
 \end{aligned}$$

To change to dB/km, multiply by  $10 \log e = 4.343$ :  $\alpha_{\text{scat}} = 3.8 \text{ dB/km}$

From Eq. (3-4b):

$$\alpha_{\text{scat}} = \frac{8\pi^3}{3\lambda^4} n^8 p^2 k_B T_f \beta_T = 1.16 \text{ km}^{-1} = 5.0 \text{ dB/km}$$

3-8. Plot of Eq. (3-7).

3-9. Plot of Eq. (3-9).

3-10. From Fig. 2-22, we make the estimates given in this table:

$v_m$	$P_{\text{clad}}/P$	$\alpha_{v_m} = \alpha_1 + (\alpha_2 - \alpha_1)P_{\text{clad}}/P$	$5 + 10^3 P_{\text{clad}}/P$
01	0.02	$3.0 + 0.02$	$5 + 20 = 25$
11	0.05	$3.0 + 0.05$	$5 + 50 = 55$
21	0.10	$3.0 + 0.10$	$5 + 100 = 105$
02	0.16	$3.0 + 0.16$	$5 + 160 = 165$
31	0.19	$3.0 + 0.19$	$5 + 190 = 195$
12	0.31	$3.0 + 0.31$	$5 + 310 = 315$

3-11. (a) We want to solve Eq. (3-12) for  $\alpha_{gi}$ . With  $\alpha = 2$  in Eq. (2-78) and letting

$$\Delta = \frac{n^2(0) - n_2^2}{2n^2(0)}$$

we have

$$\alpha(r) = \alpha_1 + (\alpha_2 - \alpha_1) \frac{n^2(0) - n^2(r)}{n^2(0) - n_2^2} = \alpha_1 + (\alpha_2 - \alpha_1) \frac{r^2}{a^2}$$

Thus

$$\alpha_{gi} = \frac{\int_0^\infty \alpha(r) p(r) r dr}{\int_0^\infty p(r) r dr} = \alpha_1 + \frac{(\alpha_2 - \alpha_1)}{a^2} \frac{\int_0^\infty \exp(-Kr^2) r^3 dr}{\int_0^\infty \exp(-Kr^2) r dr}$$

To evaluate the integrals, let  $x = Kr^2$ , so that  $dx = 2Krd r$ . Then

$$\frac{\int_0^\infty \exp(-Kr^2) r^3 dr}{\int_0^\infty \exp(-Kr^2) r dr} = \frac{\frac{1}{2K^2} \int_0^\infty e^{-x} x dx}{\frac{1}{2K} \int_0^\infty e^{-x} dx} = \frac{\frac{1}{K} 1!}{0!} = \frac{1}{K}$$

$$\text{Thus } \alpha_{gi} = \alpha_1 + \frac{(\alpha_2 - \alpha_1)}{Ka^2}$$

$$(b) \quad p(a) = 0.1 P_0 = P_0 e^{-Ka^2} \text{ yields } e^{Ka^2} = 10.$$

From this we have  $Ka^2 = \ln 10 = 2.3$ . Thus

$$\alpha_{gi} = \alpha_1 + \frac{(\alpha_2 - \alpha_1)}{2.3} = 0.57\alpha_1 + 0.43\alpha_2$$

3-12. With  $\lambda$  in units of micrometers, we have

$$n = \left\{ 1 + \frac{196.98}{(13.4)^2 - (1.24/\lambda)^2} \right\}^{1/2}$$

To compare this with Fig. 3-12, calculate three representative points, for example,  $\lambda = 0.2, 0.6$ , and  $1.0 \mu\text{m}$ . Thus we have the following:

Wavelength $\lambda$	Calculated n	n from Fig. 3-12
0.2 $\mu\text{m}$	1.548	1.550
0.6 $\mu\text{m}$	1.457	1.458
1.0 $\mu\text{m}$	1.451	1.450

- 3-13. (a) From Fig. 3-13,  $\frac{d\tau}{d\lambda} \approx 80 \text{ ps}/(\text{nm-km})$  at 850 nm. Therefore, for the LED we have from Eq. (3-20)

$$\frac{\sigma_{\text{mat}}}{L} = \frac{d\tau}{d\lambda} \sigma_{\lambda} = [80 \text{ ps}/(\text{nm-km})](45 \text{ nm}) = 3.6 \text{ ns/km}$$

For a laser diode,

$$\frac{\sigma_{\text{mat}}}{L} = [80 \text{ ps}/(\text{nm-km})](2 \text{ nm}) = 0.16 \text{ ns/km}$$

- (b) From Fig. 3-13,  $\frac{d\tau_{\text{mat}}}{d\lambda} = 22 \text{ ps}/(\text{nm-km})$

Therefore,  $D_{\text{mat}}(\lambda) = [22 \text{ ps}/(\text{nm-km})](75 \text{ nm}) = 1.65 \text{ ns/km}$

- 3-14. (a) Using Eqs. (2-48), (2-49), and (2-57), Eq. (3-21) becomes

$$b = 1 - \left( \frac{ua}{V} \right)^2 = 1 - \frac{u^2 a^2}{u^2 a^2 + w^2 a^2} = \frac{w^2}{u^2 + w^2}$$

$$= \frac{\beta^2 - k^2 n_2^2}{k^2 n_1^2 - \beta^2 + \beta^2 - k^2 n_2^2} = \frac{\beta^2 / k^2 - n_2^2}{n_1^2 - n_2^2}$$

- (b) Expand b as  $b = \frac{(\beta/k + n_2)(\beta/k - n_2)}{(n_1 + n_2)(n_1 - n_2)}$

Since  $n_2 < \beta/k < n_1$ , let  $\beta/k = n_1(1 - \delta)$  where  $0 < \delta < \Delta \ll 1$ . Thus,