10

Project appraisal and risk

Topic list	Syllabus reference
1 Risk and uncertainty	D5 (a)
2 Sensitivity analysis	D5 (b)
3 Probability analysis	D5 (c)
4 Other risk adjustment techniques	D5 (d)

Introduction

This chapter will show some of the different methods of assessing and taking account of the **risk** and **uncertainty** associated with a project. The next chapter of this Study Text will consider two further project appraisal topics – **capital rationing** and **leasing**.

Study guide

		Intellectual level
D5	**Adjusting for risk and uncertainty in investment appraisal**	
(a)	Describe and discuss the difference between risk and uncertainty in relation to probabilities and increasing project life.	2
(b)	Apply sensitivity analysis to investment projects and discuss the usefulness of sensitivity analysis in assisting investment decisions.	2
(c)	Apply probability analysis to investment projects and discuss the usefulness of probability analysis in assisting investment decisions.	2
(d)	Apply and discuss other techniques of adjusting for risk and uncertainty in investment appraisal, including:	
(i)	Simulation	1
(ii)	Adjusted payback	1
(iii)	Risk-adjusted discount rates	2

Exam guide

Risk and uncertainty are increasingly examinable in financial management exams and sensitivity calculations are particularly important. You will need to be able to explain these techniques as well as be confident and competent with the calculations.

1 Risk and uncertainty 12/07

FAST FORWARD

Risk can be applied to a situation where there are several possible outcomes and, on the basis of past relevant experience, probabilities can be assigned to the various outcomes that could prevail.

Uncertainty can be applied to a situation where there are several possible outcomes but there is little past relevant experience to enable the probability of the possible outcomes to be predicted.

There are a wide range of techniques for incorporating risk into project appraisal.

A distinction should be made between the terms risk and uncertainty.

Risk	• Several possible outcomes • On basis of past relevant experience, assign probabilities to outcomes • Increases as the variability of returns increases
Uncertainty	• Several possible outcomes • Little past experience, thus difficult to assign probabilities to outcomes • Increases as project life increases

A risky situation is one where we can say that there is a 70% probability that returns from a project will be in excess of $100,000 but a 30% probability that returns will be less than $100,000. If, however, no information can be provided on the returns from the project, we are faced with an uncertain situation.

In general, risky projects are those whose future cash flows, and hence the project returns, are likely to be variable. The greater the variability is, the greater the risk. The problem of risk is more acute with capital investment decisions than other decisions for the following reasons.

(a) **Estimates** of capital expenditure might be for **several years ahead**, such as for major construction projects. Actual costs may escalate well above budget as the work progresses.

(b) Estimates of **benefits** will be for **several years ahead**, sometimes 10, 15 or 20 years ahead or even longer, and such long-term estimates can at best be approximations.

Exam focus point

Make sure you can explain the difference between risk and uncertainty. This was required in December 2007 and the examiner commented that it caused difficulties.

2 Sensitivity analysis 12/07

FAST FORWARD

Sensitivity analysis assesses how responsive the project's NPV is to changes in the variables used to calculate that NPV. One particular approach to sensitivity analysis, the certainty-equivalent approach, involves the conversion of the expected cash flows of the project to riskless equivalent amounts.

Key term

Sensitivity analysis is one method of analysing the risk surrounding a capital expenditure project and enables an assessment to be made of how responsive the project's NPV is to changes in the variables that are used to calculate that NPV.

The NPV could depend on a number of uncertain independent variables.

- Selling price
- Sales volume
- Cost of capital
- Initial cost
- Operating costs
- Benefits

The basic approach of sensitivity analysis is to **calculate the project's NPV** under **alternative assumptions** to determine how sensitive it is to changing conditions. An indication is thus provided of those variables to which the NPV is most sensitive (**critical variables**) and the **extent** to which those variables **may change** before the investment results in a negative NPV.

Sensitivity analysis therefore provides an indication of why a project might fail. Management should review critical variables to assess whether or not there is a strong possibility of events occurring which will lead to a negative NPV. Management should also pay particular attention to controlling those variables to which the NPV is particularly sensitive, once the decision has been taken to accept the investment.

A simple approach to deciding which variables the NPV is particularly sensitive to is to calculate the sensitivity of each variable:

$$\text{Sensitivity} = \frac{\text{NPV}}{\text{Present value of project variable}} \%$$

The lower the percentage, the more sensitive is NPV to that project variable as the variable would need to change by a smaller amount to make the project non-viable.

2.1 Example: Sensitivity analysis

Kenney Co is considering a project with the following cash flows.

Year	*Initial investment*	*Variable costs*	*Cash inflows*	*Net cash flows*
	$'000	$'000	$'000	$'000
0	7,000			
1		(2,000)	6,500	4,500
2		(2,000)	6,500	4,500

Cash flows arise from selling 650,000 units at $10 per unit. Kenney Co has a cost of capital of 8%.

Required

Measure the sensitivity of the project to changes in variables.

Solution

The PVs of the cash flow are as follows.

Year	Discount factor 8%	PV of initial investment $'000	PV of variable costs $'000	PV of cash inflows $'000	PV of net cash flow $'000
0	1.000	(7,000)			(7,000)
1	0.926		(1,852)	6,019	4,167
2	0.857		(1,714)	5,571	3,857
		(7,000)	(3,566)	11,590	1,024

The project has a positive NPV and would appear to be worthwhile. The sensitivity of each project variable is as follows.

(a) **Initial investment**

$$\text{Sensitivity} = \frac{1{,}024}{7{,}000} \times 100 = 14.6\%$$

(b) **Sales volume**

$$\text{Sensitivity} = \frac{1{,}024}{11{,}590 - 3{,}566} \times 100 = 12.8\%$$

(c) **Selling price**

$$\text{Sensitivity} = \frac{1{,}024}{11{,}590} \times 100 = 8.8\%$$

(d) **Variable costs**

$$\text{Sensitivity} = \frac{1{,}024}{3{,}566} \times 100 = 28.7\%$$

(e) **Cost of capital.** We need to calculate the IRR of the project. Let us try discount rates of 15% and 20%.

Year	Net cash flow $'000	Discount factor 15%	PV $'000	Discount factor 20%	PV $'000
0	(7,000)	1	(7,000)	1	(7,000)
1	4,500	0.870	3,915	0.833	3,749
2	4,500	0.756	3,402	0.694	3,123
			NPV = 317		NPV = (128)

$$\text{IRR} = 0.15 + \left[\frac{317}{317 + 128} \times (0.20 - 0.15)\right] = 18.56\%$$

The cost of capital can therefore increase by 132% before the NPV becomes negative.

The elements to which the NPV appears to be most sensitive are the selling price followed by the sales volume. Management should thus pay particular attention to these factors so that they can be carefully monitored.

2.2 Weaknesses of this approach to sensitivity analysis

These are as follows.

(a) The method requires that **changes** in each key variable are **isolated.** However management is more interested in the combination of the effects of changes in two or more key variables.

(b) Looking at factors in isolation is unrealistic since they are often **interdependent**.

(c) Sensitivity analysis does not examine the **probability** that any particular variation in costs or revenues might occur.

(d) **Critical factors** may be those over which managers have no control.

(e) In itself it does not provide a decision rule. Parameters defining **acceptability** must be laid down by managers.

Question — Sensitivity analysis

Nevers Ure Co has a cost of capital of 8% and is considering a project with the following 'most-likely' cash flows.

Year	*Purchase of plant*	*Running costs*	*Savings*
	$	$	$
0	(7,000)		
1		2,000	6,000
2		2,500	7,000

Required

Measure the sensitivity (in percentages) of the project to changes in the levels of expected costs and savings.

Answer

The PVs of the cash flows are as follows.

Year	*Discount factor 8%*	*PV of plant cost*	*PV of running costs*	*PV of savings*	*PV of net cash flow*
		$	$	$	$
0	1.000	(7,000)			(7,000)
1	0.926		(1,852)	5,556	3,704
2	0.857		(2,143)	5,999	3,856
		(7,000)	(3,995)	11,555	560

The project has a positive NPV and would appear to be worthwhile. Sensitivity of the project to changes in the levels of expected costs and savings is as follows.

(a) **Plant costs sensitivity** = $\frac{560}{7,000} \times 100 = 8\%$

(b) **Running costs sensitivity** = $\frac{560}{3,995} \times 100 = 14\%$

(c) **Savings sensitivity** = $\frac{560}{11,555} \times 100 = 4.8\%$

Exam focus point

Examiners have commented that sensitivity analysis is often confused with the internal rate of return.

2.3 The certainty-equivalent approach

Another method is the **certainty-equivalent approach**. By this method, the expected cash flows of the project are **converted to riskless equivalent amounts**. The greater the risk of an expected cash flow, the smaller the certainty-equivalent value (for receipts) or the larger the certainty equivalent value (for payments).

As the cash flows are reduced to supposedly certain amounts, they should be discounted at a **risk free** rate. This concept will be covered in detail later in this text, but the risk free rate is effectively the level of return that can be obtained from undertaking no risk.

2.4 Example: Certainty-equivalent approach

Dark Ages Co, whose cost of capital is 10%, is considering a project with the following expected cash flows.

Year	*Cash flow*	*Discount factor 10%*	*Present value*
	$		$
0	(10,000)	1.000	(10,000)
1	7,000	0.909	6,363
2	5,000	0.826	4,130
3	5,000	0.751	3,755
			NPV = +4,248

The project seems to be worthwhile. However, because of the uncertainty about the future cash receipts, the management decides to reduce them to 'certainty-equivalents' by taking only 70%, 60% and 50% of the years 1, 2 and 3 cash flows respectively. The risk free rate is 5%.

On the basis of the information set out above, assess whether the project is worthwhile.

Solution

The risk-adjusted NPV of the project is as follows.

Year	*Cash flow*	*Discount factor 5%*	*Present value*
	$		$
0	(10,000)	1.000	(10,000)
1	4,900	0.952	4,665
2	3,000	0.907	2,721
3	2,500	0.864	2,160
			NPV = (454)

The project is too risky and should be rejected.

The disadvantage of the 'certainty-equivalent' approach is that the amount of the adjustment to each cash flow is decided **subjectively**.

3 Probability analysis 12/07, 6/10

FAST FORWARD

A **probability analysis** of expected cash flows can often be estimated and used both to calculate an expected NPV and to measure risk.

A **probability distribution** of '**expected cash flows**' can often be estimated, recognising there are several possible outcomes, not just one. This may be used to do the following.

Step 1 Calculate an expected value of the NPV

Step 2 Measure risk, for example in the following ways.

(a) By calculating the worst possible outcome and its probability.
(b) By calculating the probability that the project will fail to achieve a positive NPV.
(c) By calculating the standard deviation of the NPV.

3.1 Example: Probability estimates of cash flows

A company is considering a project involving the outlay of $300,000 which it estimates will generate cash flows over its two year life at the probabilities shown in the following table.

Cash flows for project

Year 1	*Cash flow*	*Probability*
	$	
	100,000	0.25
	200,000	0.50
	300,000	0.25
		1.00

Year 2

If cash flow in Year 1 is:	*there is a probability of:*	*that the cash flow in Year 2 will be:*
$		$
100,000	0.25	Nil
	0.50	100,000
	0.25	200,000
	1.00	
200,000	0.25	100,000
	0.50	200,000
	0.25	300,000
	1.00	
300,000	0.25	200,000
	0.50	300,000
	0.25	350,000
	1.00	

The company's cost of capital for this type of project is 10% DCF.

You are required to calculate the expected value (EV) of the project's NPV and the probability that the NPV will be negative.

Solution

Step 1 Calculate expected value of the NPV.

First we need to draw up a probability distribution of the expected cash flows. We begin by calculating the present values of the cash flows.

Year	*Cash flow*	*Discount factor 10%*	*Present value*
	$'000		$'000
1	100	0.909	90.9
1	200	0.909	181.8
1	300	0.909	272.7
2	100	0.826	82.6
2	200	0.826	165.2
2	300	0.826	247.8
2	350	0.826	289.1

Year 1 PV of cash flow $'000 (a)	*Probability* (b)	*Year 2 PV of cash flow* $'000 (c)	*Probability* (d)	*Joint probability* (b) × (d)	*Total PV of cash inflows* $'000 (a) + (c)	*EV of PV of cash inflows* $'000
90.9	0.25	0.0	0.25	0.0625	90.9	5.681
90.9	0.25	82.6	0.50	0.1250	173.5	21.688
90.9	0.25	165.2	0.25	0.0625	256.1	16.006
181.8	0.50	82.6	0.25	0.1250	264.4	33.050
181.8	0.50	165.2	0.50	0.2500	347.0	86.750
181.8	0.50	247.8	0.25	0.1250	429.6	53.700
272.7	0.25	165.2	0.25	0.0625	437.9	27.369
272.7	0.25	247.8	0.50	0.1250	520.5	65.063
272.7	0.25	289.1	0.25	0.0625	561.8	35.113
						344.420

	$
EV of PV of cash inflows	344,420
Less project cost	300,000
EV of NPV	44,420

Step 2 Measure risk.

Since the EV of the NPV is positive, the project should go ahead unless the risk is unacceptably high. The probability that the project will have a negative NPV is the probability that the total PV of cash inflows is less than $300,000. From the column headed 'Total PV of cash inflows', we can establish that this probability is 0.0625 + 0.125 + 0.0625 + 0.125 = 0.375 or 37.5%. This might be considered an unacceptably high risk.

3.2 Example: Probability estimates of cash flows excluding discounting

A company has an overdraft limit of $500,000. There are concerns that this limit may be exceeded within the next two years. As a result cash flow forecasts with their associated probabilities have been prepared for the next two periods as follows.

The cash balance at the beginning of year 1 is $500,000.

What are the expected cash balances at the end of each year and what are the probabilities of exceeding the overdraft limit at the end of each year?

Year 1cash flow $'000	*Probability*	*Year 2cash flow* $'000	*Probability*
4,000	0.20	2,500	0.10
3,000	0.50	1,000	0.65
(1,500)	0.30	(3,500)	0.25

Solution

Opening balance $'000	*Year 1 cash flow* $'000	*Closing balance for year 1* $'000	*Probability*	*Expected value* $'000
500	4,000	4,500	0.20	900
500	3,000	3,500	0.50	1,750
500	(1,500)	(1,000)	0.30	(300)
				2,350

The expected value of the cash balance at the end of year 1 is $2,350,000.
There is a 0.3 or 30% chance that the overdraft limit will be exceeded.

Year 1 closing balance	Probability	Year 2 cash flow	Probability	Year 2 closing balance	Joint probability	Expected value
$'000		$'000		$'000		$'000
(a)	(b)	(c)	(d)	(a) + (c)	(b) × (d)	
4,500	0.20	2,500	0.10	7,000	0.020	140.0
4,500	0.20	1,000	0.65	5,500	0.130	715.0
4,500	0.20	(3,500)	0.25	1,000	0.050	50.0
3,500	0.50	2,500	0.10	6,000	0.050	300.0
3,500	0.50	1,000	0.65	4,500	0.325	462.5
3,500	0.50	(3,500)	0.25	0	0.125	0.0
(1,000)	0.30	2,500	0.10	1,500	0.030	45.0
(1,000)	0.30	1,000	0.65	0	0.195	0.0
(1,000)	0.30	(3,500)	0.25	(4,500)	0.075	(337.5)
						2,375.0

The expected value of the cash balance at the end of year 2 is $2,375,000.
There is a 0.075 or 7.5% chance that the overdraft limit will be exceeded.

3.3 Problems with expected values

There are the following problems with using expected values in making investment decisions.

- An investment may be **one-off**, and 'expected' NPV may never actually occur
- **Assigning probabilities** to events is highly **subjective**
- Expected values **do not evaluate the range** of possible NPV outcomes

4 Other risk adjustment techniques

4.1 Simulation

FAST FORWARD

Other risk adjustment techniques include the use of simulation models, adjusted payback and risk-adjusted discount rates.

Simulation will overcome problems of having a very large number of possible outcomes, also the correlation of cash flows (a project which is successful in its early years is more likely to be successful in its later years).

4.2 Example: Simulation model

The following probability estimates have been prepared for a proposed project.

	Year	*Probability*	$
Cost of equipment	0	1.00	(40,000)
Revenue each year	1–5	0.15	40,000
		0.40	50,000
		0.30	55,000
		0.15	60,000
Running costs each year	1–5	0.10	25,000
		0.25	30,000
		0.35	35,000
		0.30	40,000

The cost of capital is 12%. Assess how a simulation model might be used to assess the project's NPV.

Solution

A simulation model could be constructed by assigning a range of random number digits to each possible value for each of the uncertain variables. The random numbers must exactly match their respective probabilities. This is achieved by working upwards cumulatively from the lowest to the highest cash flow values and assigning numbers that will correspond to probability groupings as follows.

Revenue				*Running costs*		
$	*Prob*	*Random numbers*		$	*Prob*	*Random numbers*
40,000	0.15	00 – 14	*	25,000	0.10	00 – 09
50,000	0.40	15 – 54	**	30,000	0.25	10 – 34
55,000	0.30	55 – 84	***	40,000	0.35	35 – 69
60,000	0.15	85 – 99		40,000	0.30	70 – 99

* Probability is 0.15 (15%). Random numbers are 15% of range 00 – 99.

** Probability is 0.40 (40%). Random numbers are 40% of range 00 – 99 but starting at 15.

*** Probability is 0.30 (30%). Random numbers are 30% of range 00 – 99 but starting at 55.

For revenue, the selection of a random number in the range 00 and 14 has a probability of 0.15. This probability represents revenue of $40,000. Numbers have been assigned to cash flows so that when numbers are selected at random, the cash flows have exactly the same probability of being selected as is indicated in their respective probability distribution above.

Random numbers would be generated, for example by a computer program, and these would be used to assign values to each of the uncertain variables.

For example, if random numbers 37, 84, 20, 01, 56 and 89 were generated, the values assigned to the variables would be as follows.

	Revenue		*Costs*	
Calculation	*Random number*	*Value*	*Random number*	*Value*
		$		$
1	37	50,000	84	40,000
2	20	50,000	01	25,000
3	56	55,000	89	40,000

A computer would calculate the NPV may times over using the values established in this way with more random numbers, and the results would be analysed to provide the following.

(a) An **expected NPV** for the project

(b) A **statistical distribution** pattern for the possible variation in the NPV above or below this average

The decision whether to go ahead with the project would then be made on the basis of **expected return** and **risk.**

4.3 Adjusted payback (discounted payback)

The payback method of investment appraisal, discussed in Chapter 7, recognises uncertainty in investment decisions by focusing on the near future. Short-term projects are preferred to long-term projects and liquidity is emphasised.

Adjusted payback uses **discounted cash flows**. This is also known as discounted payback.

One way of dealing with risk is to **shorten** the payback period required. A **maximum payback period** can be set to reflect the fact that risk increases the longer the time period under consideration. However, the disadvantages of payback as an investment appraisal method (discussed in Section 4.2 of Chapter 7) mean that adjusted payback cannot be recommended as a method of adjusting for risk.

4.4 Risk-adjusted discount rates

Investors want higher returns for higher risk investments. The greater the risk attached to future returns, the greater the risk premium required. Investors also prefer cash now to later and require a higher return for longer time periods.

In investment appraisal, a **risk-adjusted discount rate** can be used for particular types or **risk classes** of investment projects to reflect their relative risks. For example, a **high discount rate** can be used so that a cash flow which occurs quite some time in the future will have less effect on the decision. Alternatively, with the launch of a new product, a higher **initial** risk premium may be used with a decrease in the discount rate as the product becomes established.

We will study risk-adjusted discount rates in more detail in Chapter 18.

Chapter Roundup

- **Risk** can be applied to a situation where there are several possible outcomes and, on the basis of past relevant experience, probabilities can be assigned to the various outcomes that could prevail.

  Uncertainty can be applied to a situation where there are several possible outcomes but there is little past relevant experience to enable the probability of the possible outcomes to be predicted.

  There are a wide range of techniques for incorporating risk into project appraisal.

- **Sensitivity analysis** assesses how responsive the project's NPV is to changes in the variables used to calculate that NPV. One particular approach to sensitivity analysis, the certainty-equivalent approach, involves the conversion of the expected cash flows of the project to riskless equivalent amounts.
- A **probability analysis** of expected cash flows can often be estimated and used both to calculate an expected NPV and to measure risk.
- Other risk adjustment techniques include the use of simulation models, adjusted payback and risk-adjusted discount rates.

Quick Quiz

1 Give three examples of uncertain independent variables upon which the NPV of a project may depend.

2 How are simulation models constructed?

3 Describe in a sentence each three ways in which managers can reduce risk.

4 Sensitivity analysis allows for uncertainty in project appraisal by assessing the probability of changes in the decision variables.

True ☐

False ☐

5 *Fill in the blanks.*

The .... is where expected cash flows are converted to riskless equivalent amounts.

6 Give two examples of ways that risk can be measured in probability analysis.

7 Expected values can help an accountant evaluate the range of possible net present value outcomes.

True ☐

False ☐

Answers to Quick Quiz

1 (a) Selling price
  (b) Sales volume
  (c) Cost of capital
  (d) Initial cost
  (e) Operating costs
  (f) Benefits

2 By assigning a range of random number digits to each possible value of each of the uncertain variables.

3 (a) Set maximum payback period.
  (b) Use high discounting rate.
  (c) Use sensitivity analysis to determine the critical factors within the decision-making process.
  (d) Use pessimistic estimates.

4 False. Sensitivity analysis does not assess probability.

5 Certainty-equivalent approach.

6 Calculating the worst possible outcome and its probability.
  Calculating the probability that the project will fail to achieve a positive NPV.

7 False

Now try the questions below from the Exam Question Bank

Number	Level	Marks	Time
Q15	Introductory	n/a	30 mins