Systems of Equations

CHAPTER

8

Real-World Application

To stimulate the economy in his town of Brewton, Alabama, in 2009, Danny Cottrell, co-owner of The Medical Center Pharmacy, gave each of his full-time employees \$700 and each part-time employee \$300. He asked that each person donate 15% to a charity of his or her choice and spend the rest locally. The money was paid in \$2 bills, a rarely used currency, so that the business community could easily see how the money circulated. Cottrell gave away a total of \$16,000 to his 24 employees. How many full-time employees and how many part-time employees were there?

This problem appears as Example 7 in Section 8.3.

8.1 Systems of Equations in Two Variables

OBJECTIVE

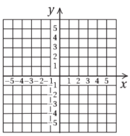
a Solve a system of two linear equations or two functions by graphing and determine whether a system is consistent or inconsistent and whether the equations in a system are dependent or independent.

SKILL TO REVIEW

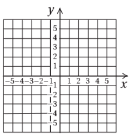
Objective 3.2a: Graph linear equations of the type $y = mx + b$ and $Ax + By = C$.

Graph.

1. $x + y = 3$



2. $y = x - 2$



We can solve many applied problems more easily by translating them to two or more equations in two or more variables than by translating to a single equation. Let's look at such a problem.

Mother's Day Spending. Mother's Day ranks fourth in spending in the United States behind the winter holidays, back-to-school buying, and Valentine's Day. About \$15.8 billion was spent to celebrate Mother's Day in 2008. Of this amount, a total of \$5 billion was spent on meals in restaurants and flowers. The amount spent on restaurant meals was \$1 billion more than the amount spent on flowers. How much was spent on each?

Source: National Retail Association

To solve, we first let

x = the amount spent on restaurant meals, and

y = the amount spent on flowers,

where x and y are in billions of dollars. The problem gives us two statements that can be translated to equations.

First, we consider the total amount spent on meals and flowers:

$$\underbrace{\text{Amount spent on meals}}_{x} \quad \underset{+}{\text{plus}} \quad \underbrace{\text{Amount spent on flowers}}_{y} \quad \underset{=}{\text{is}} \quad \underbrace{\text{Total amount spent}}_{5.}$$

The second statement compares the two different amounts spent:

$$\underbrace{\text{Amount spent on meals}}_{x} \quad \underset{=}{\text{is}} \quad \underbrace{\text{\$1 billion more than amount spent on flowers}}_{y\,+\,1.}$$

We have now translated the problem to a pair, or **system, of equations**:

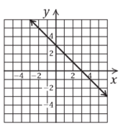
$$x + y = 5,$$
$$x = y + 1.$$

A **solution** of a system of two equations in two variables is an ordered pair that makes *both* equations true. If we graph a system of equations, the point at which the graphs intersect will be a solution of *both* equations.

Answers

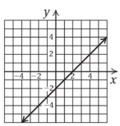
Skill to Review:

1.



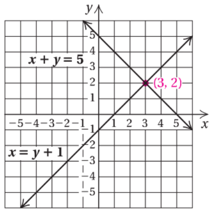
$x + y = 3$

2.



$y = x - 2$

We graph the equations listed on the preceding page.



We see that the graphs intersect at the point $(3, 2)$—that is, $x = 3$ and $y = 2$. These numbers check in the statement of the original problem. This tells us that \$3 billion was spent on restaurant meals and \$2 billion was spent on flowers.

a Solving Systems of Equations Graphically

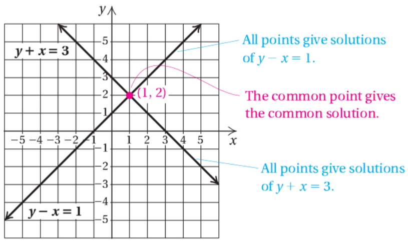
As we have just seen, we can solve systems of equations graphically.

One Solution

EXAMPLE 1 Solve this system graphically:

$$y - x = 1,$$
$$y + x = 3.$$

We draw the graph of each equation using any method studied in Chapter 3 and find the coordinates of the point of intersection.



The point of intersection has coordinates that make *both* equations true. The solution seems to be the point $(1, 2)$. However, solving by graphing may give only approximate answers. Thus we check the pair $(1, 2)$ in both equations.

Check:

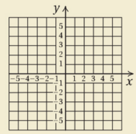
$y - x = 1$		$y + x = 3$	
$2 - 1 \; ? \; 1$		$2 + 1 \; ? \; 3$	
1	TRUE	3	TRUE

The solution is $(1, 2)$.

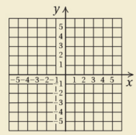
Do Exercises 1 and 2.

Solve each system graphically.

1. $-2x + y = 1,$
$3x + y = 1$



2. $y = \frac{1}{2}x,$
$y = -\frac{1}{4}x + \frac{3}{2}$



Answers

1. $(0, 1)$ **2.** $(2, 1)$

Calculator Corner

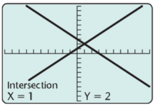
Solving Systems of Equations We can solve a system of two equations in two variables using a graphing calculator. Consider the system of equations in Example 1:

$$y - x = 1,$$
$$y + x = 3.$$

First, we solve the equations for y, obtaining $y = x + 1$ and $y = -x + 3$. Next, we enter $y_1 = x + 1$ and $y_2 = -x + 3$ on the equation-editor screen and graph the equations. We can use the standard viewing window, $[-10, 10, -10, 10]$.

We will use the INTERSECT feature to find the coordinates of the point of intersection of the lines. To access this feature, we press 2ND CALC 5. (CALC is the second operation associated with the TRACE key.) The query "First curve?" appears on the graph screen. The blinking cursor is positioned on the graph of y_1. We press ENTER to indicate that this is the first curve involved in the intersection. Next, the query "Second curve?" appears and the blinking cursor is positioned on the graph of y_2. We press ENTER to indicate that this is the second curve. Now the query "Guess?" appears. We use the ▷ and ◁ keys to move the cursor close to the point of intersection or we enter an x-value close to the first coordinate of the point of intersection. Then we press ENTER. The coordinates of the point of intersection of the graphs, $x = 1$, $y = 2$, appear at the bottom of the screen. Thus the solution of the system of equations is $(1, 2)$.

$y_1 = x + 1,\ y_2 = -x + 3$



Exercises: Use a graphing calculator to solve each system of equations.

1. $x + y = 5,$ $y = x + 1$

2. $y = x + 3,$ $2x - y = -7$

3. $x - y = -6,$ $y = 2x + 7$

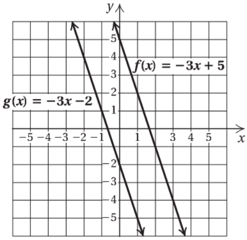
4. $x + 4y = -1,$ $x - y = 4$

No Solution

Sometimes the equations in a system have graphs that are parallel lines.

EXAMPLE 2 Solve graphically:

$$f(x) = -3x + 5,$$
$$g(x) = -3x - 2.$$



Note that this system is written using function notation. We graph the functions. The graphs have the same slope, -3, and different y-intercepts, so they are parallel. There is no point at which they cross, so the system has no solution. No matter what point we try, it will *not* check in *both* equations. The solution set is thus the empty set, denoted $\varnothing$ or $\{\ \}$.

CONSISTENT SYSTEMS AND INCONSISTENT SYSTEMS

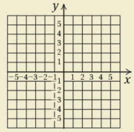
If a system of equations has at least one solution, it is **consistent**.
If a system of equations has no solution, it is **inconsistent**.

The system in Example 1 is consistent. The system in Example 2 is inconsistent.

Do Exercises 3 and 4.

3. Solve graphically:

$$y + 2x = 3,$$
$$y + 2x = -4.$$



4. Classify each of the systems in Margin Exercises 1–3 as consistent or inconsistent.

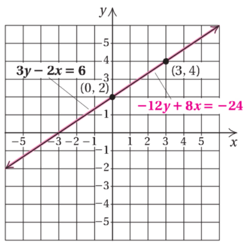
Infinitely Many Solutions

Sometimes the equations in a system have the same graph. In such a case, the equations have an *infinite* number of solutions in common.

EXAMPLE 3 Solve graphically:

$$3y - 2x = 6,$$
$$-12y + 8x = -24.$$

We graph the equations and see that the graphs are the same. Thus any solution of one of the equations is a solution of the other. Each equation has an infinite number of solutions, two of which are shown on the graph.



We check one such solution, $(0, 2)$, which is the y-intercept of each equation.

Check:

$$\begin{array}{c|c} 3y - 2x & 6 \\ \hline 3(2) - 2(0) \;?\; & 6 \\ 6 - 0 & \\ 6 & \end{array} \quad \text{TRUE} \qquad \begin{array}{c|c} -12y + 8x & -24 \\ \hline -12(2) + 8(0) \;?\; & -24 \\ -24 + 0 & \\ -24 & \end{array} \quad \text{TRUE}$$

On your own, check that $(3, 4)$ is a solution of both equations. If $(0, 2)$ and $(3, 4)$ are solutions, then all points on the line containing them will be solutions. The system has an infinite number of solutions.

Answers

3. No solution **4.** Consistent: Margin Exercises 1 and 2; inconsistent: Margin Exercise 3

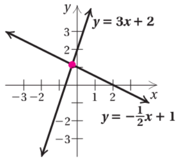
DEPENDENT EQUATIONS AND INDEPENDENT EQUATIONS

If a system of two equations in two variables:

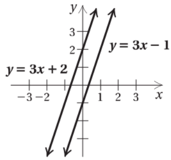
has infinitely many solutions, the equations are **dependent**.

has one solution or no solutions, the equations are **independent**.

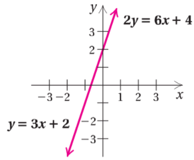
When we graph a system of two equations, one of the following three things can happen.



One solution.
Graphs intersect.
The system is consistent and *the equations are independent*.



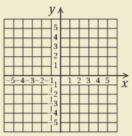
No solution.
Graphs are parallel.
The system is inconsistent and *the equations are independent*.



Infinitely many solutions.
Equations have the same graph. *The system is consistent* and *the equations are dependent*.

5. Solve graphically:

$2x - 5y = 10,$
$-6x + 15y = -30.$



6. Classify the equations in Margin Exercises 1, 2, 3, and 5 as dependent or independent.

Let's summarize what we know about the systems of equations in Examples 1–3. The system in Example 1 has exactly one solution, and the system in Example 3 has an infinite number of solutions. Since each system has at least one solution, both systems are *consistent*. The system of equations in Example 2 has no solution, so it is *inconsistent*.

The system of equations in Example 1 has exactly one solution, and the system in Example 2 has no solutions. Thus the equations in each of these systems are *independent*. In a system of equations with infinitely many solutions, the equations are dependent. This tells us that the equations in Example 3 are *dependent*. In a system with dependent equations, one equation can be obtained by multiplying the other equation by a constant.

Do Exercises 5 and 6.

Algebraic–Graphical Connection

Consider the equation $-2x + 13 = 4x - 17$. Let's solve it algebraically as we did in Chapter 2:

$$\begin{aligned} -2x + 13 &= 4x - 17 & \\ 13 &= 6x - 17 & \text{Adding } 2x \\ 30 &= 6x & \text{Adding 17} \\ 5 &= x. & \text{Dividing by 6} \end{aligned}$$

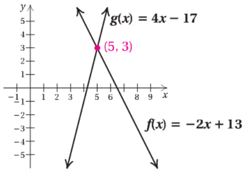
Answers

5. Infinitely many solutions
6. Independent: Margin Exercises 1, 2, and 3; dependent: Margin Exercise 5

Could we also solve the equation graphically? The answer is yes, as we see in the following two methods.

METHOD 1: Solve $-2x + 13 = 4x - 17$ graphically.

We let $f(x) = -2x + 13$ and $g(x) = 4x - 17$. Graphing the system of equations, we get the graph shown below.



The point of intersection of the two graphs is $(5, 3)$. Note that the x-coordinate of this point is 5. This is the value of x for which $-2x + 13 = 4x - 17$, so it is the solution of the equation.

Do Exercises 7 and 8.

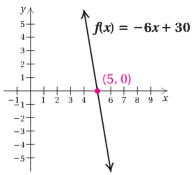
7. a) Solve $x + 1 = \frac{2}{3}x$ algebraically.

b) Solve $x + 1 = \frac{2}{3}x$ graphically using Method 1.

8. Solve $\frac{1}{2}x + 3 = 2$ graphically using Method 1.

METHOD 2: Solve $-2x + 13 = 4x - 17$ graphically.

Adding $-4x$ and 17 on both sides, we obtain an equation with 0 on one side: $-6x + 30 = 0$. This time we let $f(x) = -6x + 30$ and $g(x) = 0$. Since the graph of $g(x) = 0$, or $y = 0$, is the x-axis, we need only graph $f(x) = -6x + 30$ and see where it crosses the x-axis.



Note that the x-intercept of $f(x) = -6x + 30$ is $(5, 0)$, or just 5. This x-value is the solution of the equation $-2x + 13 = 4x - 17$.

Do Exercise 9.

9. a) Solve $x + 1 = \frac{2}{3}x$ graphically using Method 2.

b) Compare your answers to Margin Exercises 7(a), 7(b), and 9(a).

Let's compare the two methods. Using Method 1, we graph two functions. The solution of the original equation is the x-coordinate of the point of intersection. Using Method 2, we graph one function. The solution of the original equation is the x-coordinate of the x-intercept of the graph.

Do Exercise 10.

10. Solve $\frac{1}{2}x + 3 = 2$ graphically using Method 2.

Answers

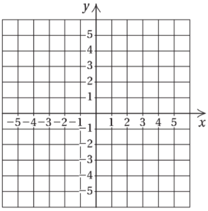
7. (a) -3; **(b)** the same: -3 **8.** -2
9. (a) -3; **(b)** All are -3. **10.** -2

8.1 Exercise Set

For Extra Help
MyMathLab
MathXL PRACTICE · WATCH · DOWNLOAD · READ · REVIEW

a Solve each system of equations graphically. Then classify the system as consistent or inconsistent and the equations as dependent or independent. Complete the check for Exercises 1–4.

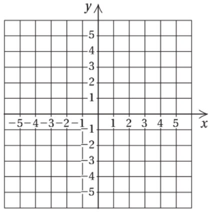
1. $x + y = 4,$
$x - y = 2$



Check: $x + y = 4$ | ?

$x - y = 2$ | ?

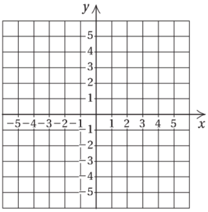
2. $x - y = 3,$
$x + y = 5$



Check: $x - y = 3$ | ?

$x + y = 5$ | ?

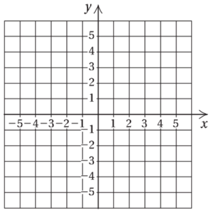
3. $2x - y = 4,$
$2x + 3y = -4$



Check: $2x - y = 4$ | ?

$2x + 3y = -4$ | ?

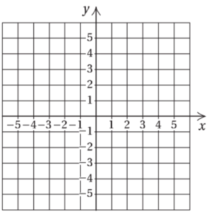
4. $3x + y = 5,$
$x - 2y = 4$



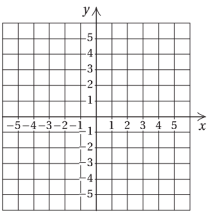
Check: $3x + y = 5$ | ?

$x - 2y = 4$ | ?

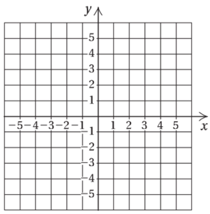
5. $2x + y = 6,$
$3x + 4y = 4$



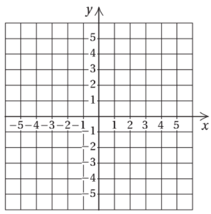
6. $2y = 6 - x,$
$3x - 2y = 6$



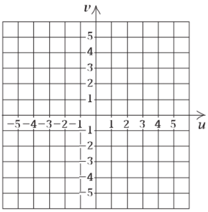
7. $f(x) = x - 1,$
$g(x) = -2x + 5$



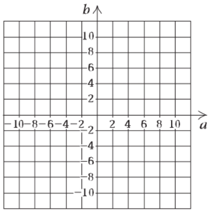
8. $f(x) = x + 1,$
$g(x) = \frac{2}{3}x$



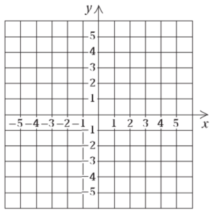
9. $2u + v = 3,$
$2u = v + 7$



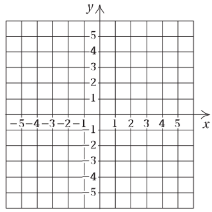
10. $2b + a = 11,$
$a - b = 5$



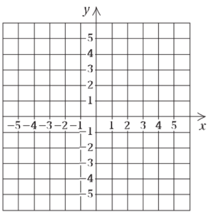
11. $f(x) = -\frac{1}{3}x - 1,$
$g(x) = \frac{4}{3}x - 6$



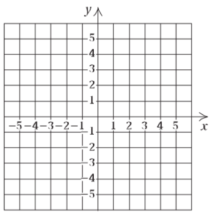
12. $f(x) = -\frac{1}{4}x + 1,$
$g(x) = \frac{1}{2}x - 2$



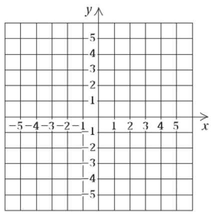
13. $6x - 2y = 2,$
$9x - 3y = 1$



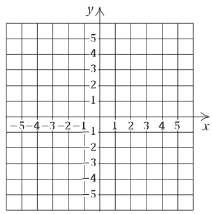
14. $y - x = 5,$
$2x - 2y = 10$



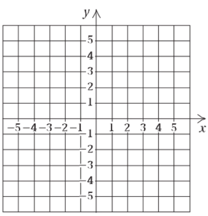
15. $2x - 3y = 6,$
$3y - 2x = -6$



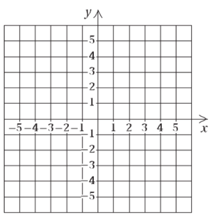
16. $y = 3 - x,$
$2x + 2y = 6$



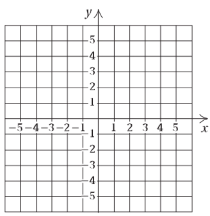
17. $x = 4,$
$y = -5$



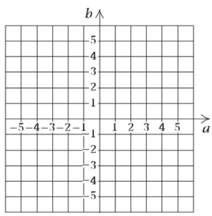
18. $x = -3,$
$y = 2$



19. $y = -x - 1,$
$4x - 3y = 17$

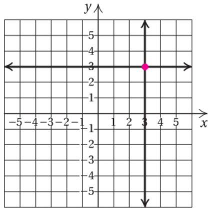


20. $a + 2b = -3,$
$b - a = 6$

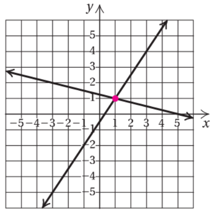


Matching. Each of Exercises 21–26 shows the graph of a system of equations and its solution. First, classify the system as consistent or inconsistent and the equations as dependent or independent. Then match it with one of the appropriate systems of equations (A)–(F), which follow.

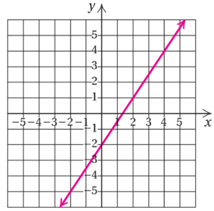
21. Solution: $(3, 3)$



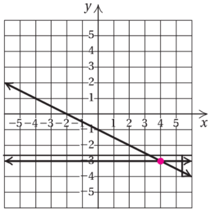
22. Solution: $(1, 1)$



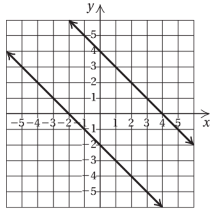
23. Solutions: Infinitely many



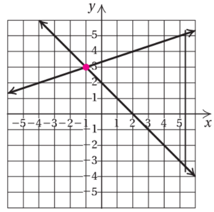
24. Solution: $(4, -3)$



25. Solution: No solution



26. Solution: $(-1, 3)$



A. $3y - x = 10,$
$x = -y + 2$

B. $9x - 6y = 12,$
$y = \frac{3}{2}x - 2$

C. $2y - 3x = -1,$
$x + 4y = 5$

D. $x + y = 4,$
$y = -x - 2$

E. $\frac{1}{2}x + y = -1,$
$y = -3$

F. $x = 3,$
$y = 3$

Skill Maintenance

Write an equation of the line containing the given point and parallel to the given line. [7.5d]

27. $(-4, 2)$; $3x = 5y - 4$

28. $(-6, 0)$; $8y - 3x = 2$

Write an equation of the line containing the given point and perpendicular to the given line. [7.5d]

29. $(-4, 6)$; $2x = 3y - 12$

30. $(3, -10)$; $8y - 4 = -6x$

Synthesis

Use a graphing calculator to solve each system of equations. Round all answers to the nearest hundredth. You may need to solve for y first.

31. $2.18x + 7.81y = 13.78,$
$5.79x - 3.45y = 8.94$

32. $f(x) = 123.52x + 89.32,$
$g(x) = -89.22x + 33.76$

Solve graphically.

33. $y = |x|,$
$x + 4y = 15$

34. $x - y = 0,$
$y = x^2$

8.2 Solving by Substitution

OBJECTIVES

a Solve systems of equations in two variables by the substitution method.

b Solve applied problems by solving systems of two equations using substitution.

SKILL TO REVIEW

Objective 2.3c: Solve equations by first removing parentheses.

Solve.

1. $3y - 4 = 5 - 3(y + 1)$

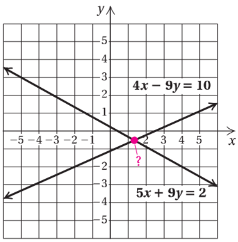
2. $2(x + 1) + 5 = 1$

Consider this system of equations:

$$5x + 9y = 2,$$
$$4x - 9y = 10.$$

What is the solution? It is rather difficult to tell exactly by graphing. It would appear that fractions are involved. It turns out that the solution is

$$\left(\frac{4}{3}, -\frac{14}{27}\right).$$



Solving by graphing, though useful in many applied situations, is not always fast or accurate in cases where solutions are not integers. We need techniques involving algebra to determine the solution exactly. Because they use algebra, they are called **algebraic methods**.

a The Substitution Method

One nongraphical method for solving systems is known as the **substitution method**.

EXAMPLE 1 Solve this system:

$$x + y = 4, \quad (1)$$
$$x = y + 1. \quad (2)$$

Equation (2) says that x and $y + 1$ name the same number. Thus we can substitute $y + 1$ for x in equation (1):

$$x + y = 4 \qquad \text{Equation (1)}$$
$$(y + 1) + y = 4. \qquad \text{Substituting } y + 1 \text{ for } x$$

Since this equation has only one variable, we can solve for y using methods learned earlier:

$$(y + 1) + y = 4$$
$$2y + 1 = 4 \qquad \text{Removing parentheses and collecting like terms}$$
$$2y = 3 \qquad \text{Subtracting 1}$$
$$y = \tfrac{3}{2}. \qquad \text{Dividing by 2}$$

We return to the original pair of equations and substitute $\frac{3}{2}$ for y in *either* equation so that we can solve for x. Calculation will be easier if we choose equation (2) since it is already solved for x:

$$x = y + 1 \qquad \text{Equation (2)}$$
$$x = \tfrac{3}{2} + 1 \qquad \text{Substituting } \tfrac{3}{2} \text{ for } y$$
$$x = \tfrac{3}{2} + \tfrac{2}{2} = \tfrac{5}{2}.$$

We obtain the ordered pair $\left(\frac{5}{2}, \frac{3}{2}\right)$. Even though we solved for y *first*, it is still the *second* coordinate since x is before y alphabetically. We check to be sure that the ordered pair is a solution.

STUDY TIPS

SKILL MAINTENANCE EXERCISES

It is never too soon to begin reviewing for the final examination. The Skill Maintenance exercises found in each exercise set review and reinforce skills taught in earlier sections. Do all of these exercises even if your instructor does not assign them. Answers to both odd-numbered exercises and even-numbered exercises appear at the back of the book.

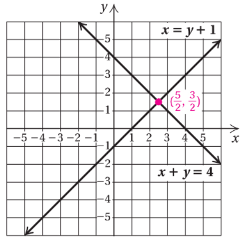
Answers

Skill to Review:
1. 1 **2.** −3

Check:

$$\begin{array}{c|c} x + y & 4 \\ \hline \frac{5}{2} + \frac{3}{2} & ?\ 4 \\ \frac{8}{2} & \\ 4 & \end{array} \quad \text{TRUE} \qquad \begin{array}{c|c} x & y + 1 \\ \hline \frac{5}{2} & ?\ \frac{3}{2} + 1 \\ & \frac{3}{2} + \frac{2}{2} \\ & \frac{5}{2} \end{array} \quad \text{TRUE}$$

Since $\left(\frac{5}{2}, \frac{3}{2}\right)$ checks, it is the solution. Even though exact fraction solutions are difficult to determine graphically, a graph can help us to visualize whether the solution is reasonable.



Solve by the substitution method.

1. $x + y = 6,$
$y = x + 2$

2. $y = 7 - x,$
$2x - y = 8$

(*Caution*: Use parentheses when you substitute, being careful about removing them. Remember to solve for both variables.)

Do Exercises 1 and 2.

Suppose neither equation of a pair has a variable alone on one side. We then solve one equation for one of the variables.

EXAMPLE 2 Solve this system:

$$2x + y = 6, \qquad (1)$$
$$3x + 4y = 4. \qquad (2)$$

First, we solve one equation for one variable. Since the coefficient of y is 1 in equation (1), it is the easier one to solve for y:

$$y = 6 - 2x. \qquad (3)$$

Next, we substitute $6 - 2x$ for y in equation (2) and solve for x:

$3x + 4(6 - 2x) = 4$	Substituting $6 - 2x$ for y
$3x + 24 - 8x = 4$	Multiplying to remove parentheses
$24 - 5x = 4$	Collecting like terms
$-5x = -20$	Subtracting 24
$x = 4.$	Dividing by -5

Caution!

Remember to use parentheses when you substitute. Then remove them properly.

In order to find y, we return to either of the original equations, (1) or (2), or equation (3), which we solved for y. It is generally easier to use an equation like (3), where we have solved for the specific variable. We substitute 4 for x in equation (3) and solve for y:

$$y = 6 - 2x = 6 - 2(4) = 6 - 8 = -2.$$

We obtain the ordered pair $(4, -2)$.

Check:

$$\begin{array}{c|c} 2x + y & 6 \\ \hline 2(4) + (-2) & ?\ 6 \\ 8 - 2 & \\ 6 & \end{array} \quad \text{TRUE} \qquad \begin{array}{c|c} 3x + 4y & 4 \\ \hline 3(4) + 4(-2) & ?\ 4 \\ 12 - 8 & \\ 4 & \end{array} \quad \text{TRUE}$$

Since $(4, -2)$ checks, it is the solution.

Do Exercises 3 and 4.

Solve by the substitution method.

3. $2y + x = 1,$
$y - 2x = 8$

4. $8x - 5y = 12,$
$x - y = 3$

Calculator Corner

Solving Systems of Equations Use the INTERSECT feature to solve the systems of equations in Margin Exercises 1–4. (See the Calculator Corner on p. 572 for the procedure.)

Answers

1. $(2, 4)$ **2.** $(5, 2)$ **3.** $(-3, 2)$
4. $(-1, -4)$

EXAMPLE 3 Solve this system of equations:

$$y = -3x + 5, \quad \textbf{(1)}$$
$$y = -3x - 2. \quad \textbf{(2)}$$

We solved this system graphically in Example 2 of Section 8.1. We found that the graphs are parallel and the system has no solution. Let's try to solve this system algebraically using substitution.

We substitute $-3x - 2$ for y in equation (1):

$$-3x - 2 = -3x + 5 \qquad \text{Substituting } -3x - 2 \text{ for } y$$
$$-2 = 5. \qquad \text{Adding } 3x$$

We have a false equation. The equation has **no solution.** (See also Example 13 of Section 2.3.)

Do Exercise 5.

5. a) Solve this system of equations algebraically using substitution:

$$y + 2x = 3,$$
$$y + 2x = -4.$$

b) Check your answer in part (a) with the one you found graphically in Margin Exercise 3 of Section 8.1.

b Solving Applied Problems Involving Two Equations

Many applied problems are easier to solve if we first translate to a system of two equations rather than to a single equation. Here we will solve a few problems that can be solved using substitution. Section 8.4 is devoted entirely to applied problems.

EXAMPLE 4 ***Architecture.*** The architects who designed the John Hancock Building in Chicago created a visually appealing building that slants on the sides. The ground floor is a rectangle that is larger than the rectangle formed by the top floor. The ground floor has a perimeter of 860 ft. The length is 100 ft more than the width. Find the length and the width.

1. **Familiarize.** We first make a drawing and label it, using l for length and w for width. We recall or look up the formula for perimeter: $P = 2l + 2w$. (This formula can be found inside the back cover of this book.)

2. **Translate.** We translate as follows:

$$\underbrace{\text{The perimeter}}_{2l + 2w} \; \underbrace{\text{is}}_{=} \; \underbrace{860 \text{ ft.}}_{860.}$$

We can also write a second equation:

$$\underbrace{\text{The length}}_{l} \; \underbrace{\text{is}}_{=} \; \underbrace{\text{100 ft more than the width.}}_{w + 100.}$$

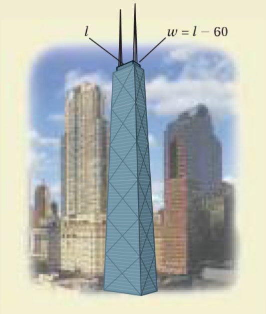
We now have a system of equations:

$$2l + 2w = 860, \quad \textbf{(1)}$$
$$l = w + 100. \quad \textbf{(2)}$$

Answers

5. (a) No solution; (b) the same—no solution

6. **Architecture.** The top floor of the John Hancock Building is also a rectangle, but its perimeter is 520 ft. The width is 60 ft less than the length. Find the length and the width.



3. **Solve.** We substitute $w + 100$ for l in equation (1):

$$
\begin{aligned}
2(w + 100) + 2w &= 860 && \text{Substituting in equation (1)}\\
2w + 200 + 2w &= 860 && \text{Multiplying to remove parentheses on the left}\\
4w + 200 &= 860 && \text{Collecting like terms}\\
\left.\begin{aligned}4w &= 660\\ w &= 165.\end{aligned}\right\} & && \text{Solving for } w
\end{aligned}
$$

Next, we substitute 165 for w in equation (2) and solve for l:

$$l = 165 + 100 = 265.$$

4. **Check.** Consider the dimensions 265 ft and 165 ft. The length is 100 ft more than the width. The perimeter is $2(265 \text{ ft}) + 2(165 \text{ ft})$, or 860 ft. The dimensions 265 ft and 165 ft check in the original problem.

5. **State.** The length is 265 ft, and the width is 165 ft.

Do Exercise 6.

STUDY TIPS

USING THIS TEXTBOOK

One of the most important ways to improve your math study skills is to learn the proper use of the textbook. Here we highlight a few points that we consider most helpful.

- **Be sure to note the special symbols a, b, c, and so on, that correspond to the objectives you are to be able to master.** The first time you see them is in the margin at the beginning of each section; the second time is in the subheadings of each section; and the third time is in the exercise set for the section. You will also find them referred to in the skill maintenance exercises in each exercise set, in the mid-chapter review, and in the review exercises at the end of the chapter, as well as in the answers to the chapter tests and the cumulative reviews. These objective symbols allow you to refer to the appropriate place in the text whenever you need to review a topic.
- **Read and study each step of each example.** The examples include important side comments that explain each step. These carefully chosen examples and notes prepare you for success in the exercise set.
- **Stop and do the margin exercises as you study a section.** Doing the margin exercises is one of the most effective ways to enhance your ability to learn mathematics from this text. Don't deprive yourself of its benefits!
- **Note the icons listed at the top of each exercise set.** These refer to the many distinctive multimedia study aids that accompany the book.
- **Odd-numbered exercises.** Usually an instructor assigns some odd-numbered exercises. When you complete these, you can check your answers at the back of the book. If you miss any, check your work in the *Student's Solutions Manual* or ask your instructor for guidance.
- **Even-numbered exercises.** Whether or not your instructor assigns the even-numbered exercises, always do some on your own. Remember, there are no answers given for the class tests, so you need to practice doing exercises without answers. Check your answers later with a friend or your instructor.

Answer

6. Length: 160 ft; width: 100 ft

8.2 Exercise Set

For Extra Help

MyMathLab

MathXL PRACTICE | WATCH | DOWNLOAD | READ | REVIEW

a Solve each system of equations by the substitution method.

1. $y = 5 - 4x,$
$2x - 3y = 13$

2. $x = 8 - 4y,$
$3x + 5y = 3$

3. $2y + x = 9,$
$x = 3y - 3$

4. $9x - 2y = 3,$
$3x - 6 = y$

5. $3s - 4t = 14,$
$5s + t = 8$

6. $m - 2n = 3,$
$4m + n = 1$

7. $9x - 2y = -6,$
$7x + 8 = y$

8. $t = 4 - 2s,$
$t + 2s = 6$

9. $-5s + t = 11,$
$4s + 12t = 4$

10. $5x + 6y = 14,$
$-3y + x = 7$

11. $2x + 2y = 2,$
$3x - y = 1$

12. $4p - 2q = 16,$
$5p + 7q = 1$

13. $3a - b = 7,$
$2a + 2b = 5$

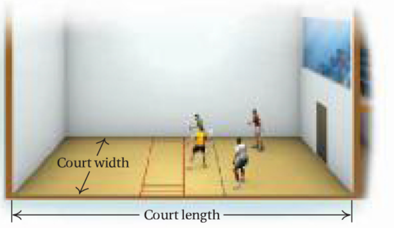
14. $5x + 3y = 4,$
$x - 4y = 3$

15. $2x - 3 = y,$
$y - 2x = 1$

16. $4x + 13y = 5,$
$-6x + y = 13$

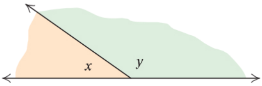
b Solve.

17. ***Racquetball Court.*** A regulation racquetball court has a perimeter of 120 ft, with a length that is twice the width. Find the length and the width of such a court.



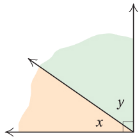
18. ***Soccer Field.*** The perimeter of a soccer field is 340 m. The length exceeds the width by 50 m. Find the length and the width.

19. ***Supplementary Angles.*** **Supplementary angles** are angles whose sum is 180°. Two supplementary angles are such that one angle is 12° less than three times the other. Find the measures of the angles.



Supplementary angles:
$x + y = 180°$

20. ***Complementary Angles.*** **Complementary angles** are angles whose sum is 90°. Two complementary angles are such that one angle is 6° more than five times the other. Find the measures of the angles.



Complementary angles:
$x + y = 90°$

21. ***Hockey Points.*** At one time, hockey teams received two points when they won a game and one point when they tied. One season, a team won a championship with 60 points. They won 9 more games than they tied. How many wins and how many ties did the team have?

22. ***Airplane Seating.*** An airplane has a total of 152 seats. The number of coach-class seats is 5 more than six times the number of first-class seats. How many of each type of seat are there on the plane?

Skill Maintenance

23. Find the slope of the line $y = 1.3x - 7$. [7.3b]

24. Simplify: $-9(y + 7) - 6(y - 4)$. [1.8b]

25. Solve $A = \dfrac{pq}{7}$ for p. [2.4b]

26. Find the slope of the line containing the points $(-2, 3)$ and $(-5, -4)$. [7.3b]

Solve. [2.3c]

27. $-4x + 5(x - 7) = 8x - 6(x + 2)$

28. $-12(2x - 3) = 16(4x - 5)$

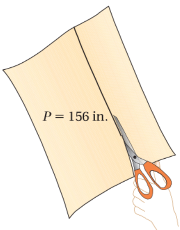
Synthesis

29. Two solutions of $y = mx + b$ are $(1, 2)$ and $(-3, 4)$. Find m and b.

30. Solve for x and y in terms of a and b:

$$5x + 2y = a,$$
$$x - y = b.$$

31. ***Design.*** A piece of posterboard has a perimeter of 156 in. If you cut 6 in. off the width, the length becomes four times the width. What were the dimensions of the original piece of posterboard?



32. ***Nontoxic Scouring Powder.*** A nontoxic scouring powder is made up of 4 parts baking soda and 1 part vinegar. How much of each ingredient is needed for a 16-oz mixture?

8.3 Solving by Elimination

OBJECTIVES

a Solve systems of equations in two variables by the elimination method.

b Solve applied problems by solving systems of two equations using elimination.

SKILL TO REVIEW

Objective 2.3b: Solve equations by first clearing the equations of fractions or decimals.

Solve. Clear fractions or decimals first.

1. $4.2x - 10.4 = 45.4 - 5.1x$

2. $\frac{1}{4}x - \frac{2}{5} + \frac{1}{2}x = \frac{3}{5} + x$

a The Elimination Method

The **elimination method** for solving systems of equations makes use of the *addition principle* for equations. Some systems are much easier to solve using the elimination method rather than the substitution method.

EXAMPLE 1 Solve this system:

$$\begin{aligned} 2x - 3y &= 0, &\quad (1)\\ -4x + 3y &= -1. &\quad (2) \end{aligned}$$

The key to the advantage of the elimination method in this case is the $-3y$ in one equation and the $3y$ in the other. These terms are opposites. If we add them, these terms will add to 0, and in effect, the variable y will have been "eliminated."

We will use the addition principle for equations, adding the same number on both sides of the equation. According to equation (2), $-4x + 3y$ and -1 are the same number. Thus we can use a vertical form and add $-4x + 3y$ to the left side of equation (1) and -1 to the right side:

$$\begin{aligned} 2x - 3y &= 0 &\quad (1)\\ \underline{-4x + 3y} &\underline{= -1} &\quad (2)\\ -2x + 0y &= -1 &\quad \text{Adding}\\ -2x + 0 &= -1 &\\ -2x &= -1. & \end{aligned}$$

We have eliminated the variable y, which is why we call this the *elimination method*.* We now have an equation with just one variable, which we solve for x:

$$\begin{aligned} -2x &= -1\\ x &= \tfrac{1}{2}. \end{aligned}$$

Next, we substitute $\frac{1}{2}$ for x in either equation and solve for y:

$$\begin{aligned} 2 \cdot \tfrac{1}{2} - 3y &= 0 &\quad \text{Substituting in equation (1)}\\ 1 - 3y &= 0 &\\ -3y &= -1 &\quad \text{Subtracting 1}\\ y &= \tfrac{1}{3}. &\quad \text{Dividing by } -3 \end{aligned}$$

We obtain the ordered pair $\left(\frac{1}{2}, \frac{1}{3}\right)$.

Check:

$2x - 3y = 0$		
$2\left(\frac{1}{2}\right) - 3\left(\frac{1}{3}\right)$	? 0	
$1 - 1$		
0		TRUE

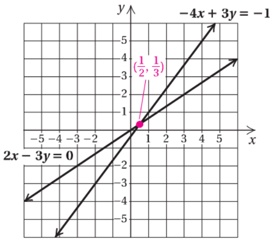
$-4x + 3y = -1$		
$-4\left(\frac{1}{2}\right) + 3\left(\frac{1}{3}\right)$	? -1	
$-2 + 1$		
-1		TRUE

*This method is also called the *addition method*.

Answers

Skill to Review:
1. 6 **2.** -4

Since $\left(\frac{1}{2}, \frac{1}{3}\right)$ checks, it is the solution. We can also see this in the graph shown at left.



Do Exercises 1 and 2.

Solve by the elimination method.

1. $5x + 3y = 17,$
$-5x + 2y = 3$

2. $-3a + 2b = 0,$
$3a - 4b = -1$

In order to eliminate a variable, we sometimes use the multiplication principle to multiply one or both of the equations by a particular number before adding.

EXAMPLE 2 Solve this system:

$$3x + 3y = 15, \quad (1)$$
$$2x + 6y = 22. \quad (2)$$

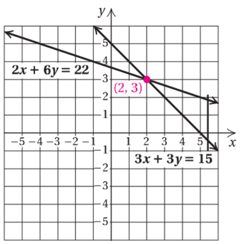
If we add directly, we get $5x + 9y = 37$, and we have not eliminated a variable. However, note that if the $3y$ in equation (1) were $-6y$, we could eliminate y. Thus we multiply by -2 on both sides of equation (1) and add:

$$
\begin{aligned}
-6x - 6y &= -30 && \text{Multiplying by } -2 \text{ on both sides of equation (1)}\\
2x + 6y &= 22 && \text{Equation (2)}\\
\hline
-4x + 0 &= -8 && \text{Adding}\\
-4x &= -8 &&\\
x &= 2. && \text{Solving for } x
\end{aligned}
$$

Then

$$
\begin{aligned}
2 \cdot 2 + 6y &= 22 && \text{Substituting 2 for } x \text{ in equation (2)}\\
\left.\begin{aligned} 4 + 6y &= 22\\ 6y &= 18\\ y &= 3. \end{aligned}\right\} && && \text{Solving for } y
\end{aligned}
$$

We obtain $(2, 3)$, or $x = 2$, $y = 3$. This checks, so it is the solution. We can also see this in the graph at left.



Do Exercise 3.

3. Solve by the elimination method:
$2y + 3x = 12,$
$-4y + 5x = -2.$

Sometimes we must multiply twice in order to make two terms opposites.

EXAMPLE 3 Solve this system:

$$2x + 3y = 17, \quad (1)$$
$$5x + 7y = 29. \quad (2)$$

We must first multiply in order to make one pair of terms with the same variable opposites. We decide to do this with the x-terms in each equation. We multiply equation (1) by 5 and equation (2) by -2. Then we get $10x$ and $-10x$, which are opposites.

$$
\begin{aligned}
\textit{From equation } (1)\text{:} \quad 10x + 15y &= 85 && \text{Multiplying by 5}\\
\textit{From equation } (2)\text{:} \quad -10x - 14y &= -58 && \text{Multiplying by } -2\\
\hline
0 + y &= 27 && \text{Adding}\\
y &= 27 && \text{Solving for } y
\end{aligned}
$$

Answers

1. $(1, 4)$ **2.** $\left(\frac{1}{3}, \frac{1}{2}\right)$ **3.** $(2, 3)$

Then

$$2x + 3 \cdot 27 = 17 \qquad \text{Substituting 27 for } y \text{ in equation (1)}$$

$$\left.\begin{aligned} 2x + 81 &= 17 \\ 2x &= -64 \\ x &= -32. \end{aligned}\right\} \qquad \text{Solving for } x$$

We check the ordered pair $(-32, 27)$.

Check:

$$\begin{array}{r|l} 2x + 3y & = 17 \\ \hline 2(-32) + 3(27) & ?\ 17 \\ -64 + 81 & \\ 17 & \quad \text{TRUE} \end{array} \qquad \begin{array}{r|l} 5x + 7y & = 29 \\ \hline 5(-32) + 7(27) & ?\ 29 \\ -160 + 189 & \\ 29 & \quad \text{TRUE} \end{array}$$

We obtain $(-32, 27)$, or $x = -32$, $y = 27$, as the solution.

Do Exercises 4 and 5.

Solve by the elimination method.

4. $4x + 5y = -8,$
$7x + 9y = 11$

5. $4x - 5y = 38,$
$7x - 8y = -22$

Some systems have no solution, as we saw graphically in Section 8.1 and algebraically in Example 3 of Section 8.2. How do we recognize such systems if we are solving using elimination?

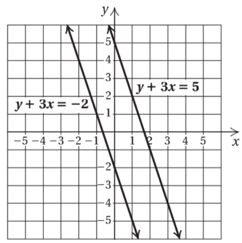
EXAMPLE 4 Solve this system:

$$y + 3x = 5, \qquad (1)$$
$$y + 3x = -2. \qquad (2)$$

If we find the slope–intercept equations for this system, we get

$$y = -3x + 5,$$
$$y = -3x - 2.$$

The graphs are parallel lines. The system has no solution.



Let's see what happens if we attempt to solve the system by the elimination method. We multiply by -1 on both sides of equation (2) and add:

$$\begin{aligned} y + 3x &= 5 && \text{Equation (1)} \\ -y - 3x &= 2 && \text{Multiplying equation (2) by } -1 \\ \hline 0 &= 7. && \text{Adding, we obtain a false equation.} \end{aligned}$$

The x-terms and the y-terms are eliminated and we have a *false* equation. Thus, if we obtain a false equation, such as $0 = 7$, when solving algebraically, we know that the system has **no solution**. The system is inconsistent, and the equations are independent.

Do Exercise 6.

6. Solve by the elimination method:

$$y + 2x = 3,$$
$$y + 2x = -1.$$

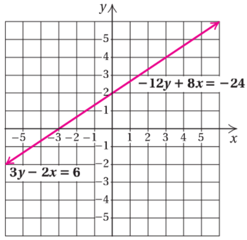
Answers

4. $(-127, 100)$ **5.** $(-138, -118)$
6. No solution

Some systems have infinitely many solutions. How can we recognize such a situation when we are solving systems using an algebraic method?

EXAMPLE 5 Solve this system:

$$3y - 2x = 6, \quad (1)$$
$$-12y + 8x = -24. \quad (2)$$



We see from the figure at left that the graphs are the same line. The system has an infinite number of solutions.

Suppose we try to solve this system by the elimination method:

$$\begin{aligned} 12y - 8x &= 24 && \text{Multiplying equation (1) by 4} \\ -12y + 8x &= -24 && \text{Equation (2)} \\ \hline 0 &= 0. && \text{Adding, we obtain a true equation.} \end{aligned}$$

We have eliminated both variables, and what remains is a true equation, $0 = 0$. It can be expressed as $0 \cdot x + 0 \cdot y = 0$, and is true for all numbers x and y. If an ordered pair is a solution of one of the original equations, then it will be a solution of the other. The system has an **infinite number of solutions**. The system is consistent, and the equations are dependent.

SPECIAL CASES

When solving a system of two linear equations in two variables:

1. If a false equation is obtained, such as $0 = 7$, then the system has no solution. The system is *inconsistent*, and the equations are *independent*.
2. If a true equation is obtained, such as $0 = 0$, then the system has an infinite number of solutions. The system is *consistent*, and the equations are *dependent*.

Do Exercise 7.

When solving a system of equations using the elimination method, it helps to first write the equations in the form $Ax + By = C$. When decimals or fractions occur, it also helps to *clear* before solving.

EXAMPLE 6 Solve this system:

$$0.2x + 0.3y = 1.7,$$
$$\tfrac{1}{7}x + \tfrac{1}{5}y = \tfrac{29}{35}.$$

We have

$$0.2x + 0.3y = 1.7, \xrightarrow{\text{Multiplying by 10 to clear decimals}} 2x + 3y = 17,$$
$$\tfrac{1}{7}x + \tfrac{1}{5}y = \tfrac{29}{35} \xrightarrow{\text{Multiplying by 35 to clear fractions}} 5x + 7y = 29.$$

We multiplied by 10 to clear the decimals. Multiplication by 35, the least common denominator, clears the fractions. The problem is now identical to Example 3. The solution is $(-32, 27)$, or $x = -32$, $y = 27$.

Do Exercises 8 and 9.

7. Solve by the elimination method:

$$2x - 5y = 10,$$
$$-6x + 15y = -30.$$

8. Clear the decimals. Then solve.

$$0.02x + 0.03y = 0.01,$$
$$0.3x - 0.1y = 0.7$$

(*Hint*: Multiply the first equation by 100 and the second one by 10.)

9. Clear the fractions. Then solve.

$$\frac{3}{5}x + \frac{2}{3}y = \frac{1}{3},$$
$$\frac{3}{4}x - \frac{1}{3}y = \frac{1}{4}$$

Answers

7. Infinitely many solutions
8. $2x + 3y = 1$,
$3x - y = 7$; $(2, -1)$
9. $9x + 10y = 5$,
$9x - 4y = 3$; $\left(\frac{25}{63}, \frac{1}{7}\right)$

To use the elimination method to solve systems of two equations:

1. Write both equations in the form $Ax + By = C$.
2. Clear any decimals or fractions.
3. Choose a variable to eliminate.
4. Make the chosen variable's terms opposites by multiplying one or both equations by appropriate numbers if necessary.
5. Eliminate a variable by adding the respective sides of the equations and then solve for the remaining variable.
6. Substitute in either of the original equations to find the value of the other variable.

Comparing Methods

When deciding which method to use, consider this table and directions from your instructor. The situation is analogous to having a piece of wood to cut and three different types of saws available. Although all three saws can cut the wood, the "best" choice depends on the particular piece of wood, the type of cut being made, and your level of skill with each saw.

METHOD	STRENGTHS	WEAKNESSES
Graphical	Can "see" solutions.	Inexact when solutions involve numbers that are not integers. Solutions may not appear on the part of the graph drawn.
Substitution	Yields exact solutions. Convenient to use when a variable has a coefficient of 1.	Can introduce extensive computations with fractions. Cannot "see" solutions quickly.
Elimination	Yields exact solutions. Convenient to use when no variable has a coefficient of 1. The preferred method for systems of 3 or more equations in 3 or more variables. (See Section 8.5.)	Cannot "see" solutions quickly.

b Solving Applied Problems Using Elimination

Let's now solve an applied problem using the elimination method. (We will solve many more problems in Section 8.4, which is devoted entirely to applied problems.)

EXAMPLE 7 *Stimulating the Hometown Economy.* To stimulate the economy in his town of Brewton, Alabama, in 2009, Danny Cottrell, co-owner of The Medical Center Pharmacy, gave each of his full-time employees \$700 and each part-time employee \$300. He asked that each person donate 15% to a charity of his or her choice and spend the rest locally. The money was paid in \$2 bills, a rarely used currency, so that the business community could easily see how the money circulated. Cottrell gave away a total of \$16,000 to his 24 employees. How many full-time employees and how many part-time employees were there?

Source: *The Press-Register,* March 4, 2009

1. **Familiarize.** We let f = the number of full-time employees and p = the number of part-time employees. Each full-time employee received \$700, so a total of $700f$ was paid to them. Similarly, the part-time employees received a total of $300p$. Thus a total of $700f + 300p$ was given away.

2. **Translate.** We translate to two equations.

$$\underbrace{\text{Total amount given away}}_{700f + 300p} \ \underset{=}{\text{is}} \ \underset{16{,}000}{\$16{,}000.}$$

$$\underbrace{\text{Total number of employees}}_{f + p} \ \underset{=}{\text{is}} \ \underset{24}{24.}$$

We now have a system of equations:

$$700f + 300p = 16{,}000, \quad (1)$$
$$f + p = 24. \quad (2)$$

3. **Solve.** First, we multiply by -300 on both sides of equation (2) and add:

$$\begin{array}{rcll} 700f + 300p &=& 16{,}000 & \text{Equation (1)} \\ -300f - 300p &=& -7200 & \text{Multiplying by } -300 \text{ on both sides of equation (2)} \\ \hline 400f &=& 8800 & \text{Adding} \\ f &=& 22. & \text{Solving for } f \end{array}$$

Next, we substitute 22 for f in equation (2) and solve for p:

$$22 + p = 24$$
$$p = 2.$$

4. **Check.** If there are 22 full-time employees and 2 part-time employees, there is a total of $22 + 2$, or 24, employees. The 22 full-time employees received a total of $\$700 \cdot 22$, or \$15,400, and the 2 part-time employees received a total of $\$300 \cdot 2$, or \$600. Then a total of \$15,400 + \$600, or \$16,000, was given away. The numbers check in the original problem.

5. **State.** There were 22 full-time employees and 2 part-time employees.

Do Exercise 10.

STUDY TIPS

FIVE STEPS FOR PROBLEM SOLVING

Remember to use the five steps for problem solving.

1. **Familiarize** yourself with the situation. Carefully read and reread the problem; draw a diagram, if appropriate; determine whether there is a formula that applies; assign letter(s), or variable(s), to the unknown(s).
2. **Translate** the problem to an equation, an inequality, or a system of equations using the variable(s) assigned.
3. **Solve** the equation, inequality, or system of equations.
4. **Check** the answer in the original wording of the problem.
5. **State** the answer clearly with appropriate units.

10. Bonuses. Monica gave each of the full-time employees in her small business a year-end bonus of \$500 while each part-time employee received \$250. She gave a total of \$4000 in bonuses to her 10 employees. How many full-time employees and how many part-time employees did Monica have?

Answer

10. Full-time: 6; part-time: 4

8.3 Exercise Set

For Extra Help
MyMathLab
MathXL PRACTICE · WATCH · DOWNLOAD · READ · REVIEW

a Solve each system of equations by the elimination method.

1. $x + 3y = 7,$
$-x + 4y = 7$

2. $x + y = 9,$
$2x - y = -3$

3. $9x + 5y = 6,$
$2x - 5y = -17$

4. $2x - 3y = 18,$
$2x + 3y = -6$

5. $5x + 3y = -11,$
$3x - y = -1$

6. $2x + 3y = -9,$
$5x - 6y = -9$

7. $5r - 3s = 19,$
$2r - 6s = -2$

8. $2a + 3b = 11,$
$4a - 5b = -11$

9. $2x + 3y = 1,$
$4x + 6y = 2$

10. $3x - 2y = 1,$
$-6x + 4y = -2$

11. $5x - 9y = 7,$
$7y - 3x = -5$

12. $5x + 4y = 2,$
$2x - 8y = 4$

13. $3x + 2y = 24,$
$2x + 3y = 26$

14. $5x + 3y = 25,$
$3x + 4y = 26$

15. $2x - 4y = 5,$
$2x - 4y = 6$

16. $3x - 5y = -2,$
$5y - 3x = 7$

17. $2a + b = 12,$
$a + 2b = -6$

18. $10x + y = 306,$
$10y + x = 90$

19. $\frac{1}{3}x + \frac{1}{5}y = 7,$
$\frac{1}{6}x - \frac{2}{5}y = -4$

20. $\frac{2}{3}x + \frac{1}{7}y = -11,$
$\frac{1}{7}x - \frac{1}{3}y = -10$

21. $\frac{1}{5}x + \frac{1}{2}y = 6,$
$\frac{2}{5}x - \frac{3}{2}y = -8$

22. $\frac{2}{3}x + \frac{3}{5}y = -17,$
$\frac{1}{2}x - \frac{1}{3}y = -1$

23. $\frac{1}{2}x - \frac{1}{3}y = -4,$
$\frac{1}{4}x + \frac{5}{6}y = 4$

24. $\frac{4}{3}x + \frac{3}{2}y = 4,$
$\frac{5}{6}x - \frac{1}{8}y = -6$

25. $0.3x - 0.2y = 4,$
$0.2x + 0.3y = 0.5$

26. $0.7x - 0.3y = 0.5,$
$-0.4x + 0.7y = 1.3$

27. $0.05x + 0.25y = 22,$
$0.15x + 0.05y = 24$

28. $1.3x - 0.2y = 12,$
$0.4x + 17y = 89$

b Solve. Use the elimination method when solving the translated system.

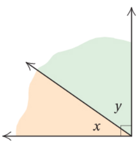
29. *Finding Numbers.* The sum of two numbers is 63. The larger number minus the smaller number is 9. Find the numbers.

30. *Finding Numbers.* The sum of two numbers is 2. The larger number minus the smaller number is 20. Find the numbers.

31. *Finding Numbers.* The sum of two numbers is 3. Three times the larger number plus two times the smaller number is 24. Find the numbers.

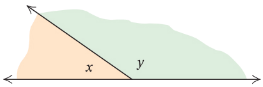
32. *Finding Numbers.* The sum of two numbers is 9. Two times the larger number plus three times the smaller number is 2. Find the numbers.

33. *Complementary Angles.* Two angles are complementary. (**Complementary angles** are angles whose sum is 90°.) Their difference is 6°. Find the angles.



Complementary angles:
$x + y = 90°$

34. *Supplementary Angles.* Two angles are supplementary. (**Supplementary angles** are angles whose sum is 180°.) Their difference is 22°. Find the angles.



Supplementary angles:
$x + y = 180°$

35. *Basketball Scoring.* Jared's Youth League basketball team scored on 27 shots, some two-point field goals and the rest one-point free throws. The team scored a total of 48 points in the game. How many of each kind of shot was made?

36. *Hockey Scoring.* At one time, hockey teams received two points when they won a game and one point when they tied. One season, a team won a championship with 65 points. They played 35 games. How many wins and how many ties did the team have?

37. ***Sales Promotion.*** Rick's Sporting Goods ran a promotion offering either a free rechargeable lantern or a free portable propane grill to each customer who bought a deluxe family tent. The store's cost for each lantern was \$20, and its cost for each grill was \$25. At the end of the promotion, 12 tents had been sold. The store's total cost for the items given away was \$280. How many of each type of free item did the customers choose?

38. ***Sales Promotion.*** The Serenity Yoga Center offered patrons who bought a 24-class pass either a free eye pillow or a free yoga DVD. The center's cost for each eye pillow was \$10, and its cost for each DVD was \$8. A total of 15 people took advantage of the offer. The center's total cost for the promotional items was \$136. How many of each item did the patrons choose?

Skill Maintenance

Given the function $f(x) = 3x^2 - x + 1$, find each of the following function values. [7.1b]

39. $f(0)$

40. $f(-1)$

41. $f(1)$

42. $f(10)$

43. $f(-2)$

44. $f(2a)$

45. $f(-4)$

46. $f(1.8)$

47. Find the domain of the function

$$f(x) = \frac{x-5}{x+7}.$$ [7.2a]

48. Find the domain and the range of the function

$$g(x) = 5 - x^2.$$ [7.2a]

49. Find an equation of the line with slope $-\frac{3}{5}$ and y-intercept $(0, -7)$. [7.5a]

50. Find an equation of the line containing the points $(-10, 2)$ and $(-2, 10)$. [7.5c]

Synthesis

51. Use the INTERSECT feature to solve the following system of equations. You may need to first solve for y. Round answers to the nearest hundredth.

$$3.5x - 2.1y = 106.2,$$
$$4.1x + 16.7y = -106.28$$

52. Solve:

$$\frac{x+y}{2} - \frac{x-y}{5} = 1,$$
$$\frac{x-y}{2} + \frac{x+y}{6} = -2.$$

53. The solution of this system is $(-5, -1)$. Find A and B.

$$Ax - 7y = -3,$$
$$x - By = -1$$

54. Find an equation to pair with $6x + 7y = -4$ such that $(-3, 2)$ is a solution of the system.

55. The points $(0, -3)$ and $\left(-\frac{3}{2}, 6\right)$ are two of the solutions of the equation $px - qy = -1$. Find p and q.

56. Determine a and b for which $(-4, -3)$ will be a solution of the system

$$ax + by = -26,$$
$$bx - ay = 7.$$

8.4 Solving Applied Problems: Two Equations

OBJECTIVES

a Solve applied problems involving total value and mixture using systems of two equations.

b Solve applied problems involving motion using systems of two equations.

a Total-Value Problems and Mixture Problems

Systems of equations can be a useful tool in solving applied problems. Using systems often makes the *Translate* step easier than using a single equation. The first kind of problem we consider involves quantities of items purchased and the total value, or cost, of the items. We refer to this type of problem as a **total-value problem**.

EXAMPLE 1 *School Lunches.* To serve lunch to students, school cafeterias receive up to \$2.47 per lunch from the U.S. government. After expenses such as labor, transportation, utilities, and equipment, schools are left with a little more than \$1 to spend on food. Of this amount, about 25 cents is spent for a carton of milk, another 25 cents for fruit and/or vegetables, and the remaining 50 cents for a main dish. In buying food for a week's lunches, one school purchased 580 servings of two menu items: the ingredients for turkey/cheese wraps at \$0.56 per serving and mixed vegetables at \$0.22 per serving. The total cost of these two menu items was \$246.60. How many servings of each type of item were purchased?

Source: *USA Today*, May 1, 2008

1. **Familiarize.** Let's begin by making a guess that the ingredients for 300 servings of turkey/cheese wraps were purchased along with 280 servings of mixed vegetables. This is a total of 580 servings. Now let's find the total cost of this order. Since the turkey/cheese wraps cost \$0.56 per serving and the vegetables cost \$0.22 per serving, the total cost would be

$$\underbrace{\$0.56(300)}_{\text{Cost of turkey/cheese wrap ingredients}} \underbrace{+}_{\text{plus}} \underbrace{\$0.22(280)}_{\text{Cost of mixed vegetables}} = \$168.00 + \$61.60$$
$$= \$229.60.$$

Although the total number of servings is correct, our guess is incorrect because the problem states that the total cost was \$246.60. Since \$229.60 is less than \$246.60, we see that more servings of the more expensive food were bought than we guessed. Nevertheless, the guess gives us useful information about how to translate this problem to a system of equations.

We let $t =$ the number of servings of turkey/cheese wrap ingredients and $v =$ the number of servings of mixed vegetables that were purchased. The ingredients for each serving of the turkey/cheese wraps cost \$0.56, so the cost of t servings is $0.56t$. Similarly, the cost for each serving of mixed vegetables is \$0.22, so the cost of v servings of mixed vegetables is $0.22v$.

It is helpful to organize the information we have in a table, as follows.

	TURKEY/CHEESE WRAPS	MIXED VEGETABLES	TOTAL	
NUMBER OF SERVINGS	t	v	580	$\rightarrow t + v = 580$
COST PER SERVING	\$0.56	\$0.22		
TOTAL COST	\$0.56t	\$0.22v	\$246.60	$\rightarrow 0.56t + 0.22v = 246.60$

2. Translate. The first row of the table gives us one equation:

$$t + v = 580.$$

The last row of the table gives us a second equation:

$$0.56t + 0.22v = 246.60.$$

We can multiply by 100 on both sides of the second equation to clear the decimals. This gives us the following system of equations:

$$t + v = 580, \qquad (1)$$

$$56t + 22v = 24{,}660. \qquad (2)$$

3. Solve. We use the elimination method to solve the system of equations. We eliminate v by multiplying by -22 on both sides of equation (1) and then adding the result to equation (2):

$$\begin{aligned} -22t - 22v &= -12{,}760 && \text{Multiplying equation (1) by } -22 \\ 56t + 22v &= 24{,}660 && \text{Equation (2)} \\ \hline 34t \qquad &= 11{,}900 && \text{Adding} \\ t &= 350. && \text{Dividing by 34} \end{aligned}$$

Next, we substitute 350 for t in equation (1) and solve for v:

$$\begin{aligned} t + v &= 580 && \text{Equation (1)} \\ 350 + v &= 580 && \text{Substituting 350 for } t \\ v &= 230. && \text{Solving for } v \end{aligned}$$

We obtain $(350, 230)$, or $t = 350$, $v = 230$.

4. Check. We check in the original problem.

Total number of servings: $t + v = 350 + 230 = 580$

Cost of turkey/cheese wraps: $\$0.56t = \$0.56(350) = \$196.00$

Cost of mixed vegetables: $\$0.22v = \$0.22(230) = \$\ \ 50.60$

Total $= \$246.60$

The numbers check.

5. State. The school bought the ingredients for 350 servings of turkey/cheese wraps and 230 servings of mixed vegetables.

Do Exercise 1.

1. Retail Sales of Sweatshirts. A campus bookstore sells college sweatshirts. White sweatshirts sell for \$18.95 each and red ones sell for \$19.50 each. If receipts for the sale of 30 sweatshirts total \$572.90, how many of each color did the shop sell?

Complete the following table, letting w = the number of white sweatshirts and r = the number of red sweatshirts.

	WHITE SWEATSHIRT	RED SWEATSHIRT	TOTAL	
NUMBER SOLD	w	r	30	$\rightarrow (\ ) + r = 30$
PRICE		\$19.50		
AMOUNT TAKEN IN	$18.95w$			$\rightarrow 18.95w + (\ ) = (\ )$

Answer

1. White: 22; red: 8

White	Red	Total	
w	r	30	$\rightarrow w + r = 30$
\$18.95	\$19.50		
$18.95w$	$19.50r$	572.90	$\rightarrow 18.95w + 19.50r = 572.90$

The following problem, similar to Example 1, is called a **mixture problem**.

EXAMPLE 2 ***Blending Flower Seeds.*** Tara's Web site, Garden Edibles, specializes in the sale of herbs and flowers for colorful meals and garnishes. Tara sells packets of nasturtium seeds for \$0.95 each and packets of Johnny-jump-up seeds for \$1.43 each. She decides to offer a 16-packet spring-garden mixture, combining packets of both types of seeds at \$1.10 per packet. How many packets of each type of seed should be put in her garden mix?

1. **Familiarize.** To familiarize ourselves with the problem situation, we make a guess and do some calculations. The total number of packets of seeds is 16. Let's try 12 packets of nasturtiums and 4 packets of Johnny-jump-ups.

   The sum of the number of packets is $12 + 4$, or 16.

   The value of these seed packets is found by multiplying the cost per packet by the number of packets and adding:

   $\$0.95(12) + \$1.43(4)$, or \$17.12.

   The desired cost is \$1.10 per packet. If we multiply \$1.10 by 16, we get $16(\$1.10)$, or \$17.60. This shows us that the guess is incorrect, but these calculations give us a basis for understanding how to translate.

   We let $a =$ the number of packets of nasturtium seeds and $b =$ the number of packets of Johnny-jump-up seeds. Next, we organize the information in a table, as follows.

	NASTURTIUM	JOHNNY-JUMP-UP	SPRING	
NUMBER OF PACKETS	a	b	16	$\rightarrow a + b = 16$
PRICE PER PACKET	\$0.95	\$1.43	\$1.10	
VALUE OF PACKETS	$0.95a$	$1.43b$	$16 \cdot 1.10$, or 17.60	$\rightarrow 0.95a + 1.43b = 17.60$

STUDY TIPS

MAKING APPLICATIONS REAL

Newspapers and magazines are full of mathematical applications. Find such an application and share it with your class. As you develop more skills in mathematics, you will find yourself observing the world from a different perspective, seeing mathematics everywhere. Math courses become more interesting when we connect the concepts to the real world.

2. **Translate.** The total number of packets is 16, so we have one equation:

$$a + b = 16.$$

The value of the nasturtium seeds is $0.95a$ and the value of the Johnny-jump-up seeds is $1.43b$. These amounts are in dollars. Since the total value is to be 16($1.10), or $17.60, we have

$$0.95a + 1.43b = 17.60.$$

We can multiply by 100 on both sides of this equation in order to clear the decimals. Thus we have translated to a system of equations:

$$a + b = 16, \qquad (1)$$
$$95a + 143b = 1760. \qquad (2)$$

3. **Solve.** We decide to use substitution, although elimination could be used as we did in Example 1. When equation (1) is solved for b, we get $b = 16 - a$. Substituting $16 - a$ for b in equation (2) and solving gives us

$$95a + 143(16 - a) = 1760 \quad \text{Substituting}$$
$$95a + 2288 - 143a = 1760 \quad \text{Using the distributive law}$$
$$-48a = -528 \quad \text{Subtracting 2288 and collecting like terms}$$
$$a = 11.$$

We have $a = 11$. Substituting this value in the equation $b = 16 - a$, we obtain $b = 16 - 11$, or 5.

4. **Check.** We check in a manner similar to our guess in the *Familiarize* step. The total number of packets is $11 + 5$, or 16. The value of the packet mixture is

$$\$0.95(11) + \$1.43(5), \text{ or } \$17.60.$$

Thus the numbers of packets check.

5. **State.** The spring-garden mixture can be made by combining 11 packets of nasturtium seeds with 5 packets of Johnny-jump-up seeds.

Do Exercise 2.

EXAMPLE 3 ***Student Loans.*** Jed's student loans totaled $16,200. Part was a Perkins loan made at 5% interest and the rest was a Stafford loan made at 4% interest. After one year, Jed's loans accumulated $715 in interest. What was the amount of each loan?

1. **Familiarize.** Listing the given information in a table will help. The columns in the table come from the formula for simple interest: $I = Prt$. We let $x =$ the number of dollars in the Perkins loan and $y =$ the number of dollars in the Stafford loan.

	PERKINS LOAN	STAFFORD LOAN	TOTAL	
PRINCIPAL	x	y	$16,200	$\rightarrow x + y = 16,200$
RATE OF INTEREST	5%	4%		
TIME	1 year	1 year		
INTEREST	$0.05x$	$0.04y$	$715	$\rightarrow 0.05x + 0.04y = 715$

2. **Blending Coffees.** The Coffee Counter charges $9.00 per pound for Kenyan French Roast coffee and $8.00 per pound for Sumatran coffee. How much of each type should be used to make a 20-lb blend that sells for $8.40 per pound?

Answer

2. Kenyan: 8 lb; Sumatran: 12 lb

3. **Client Investments.** Kaufman Financial Corporation makes investments for corporate clients. It makes an investment of \$3700 for one year at simple interest, yielding \$297. Part of the money is invested at 7% and the rest at 9%. How much was invested at each rate?

Do the *Familiarize* and *Translate* steps by completing the following table. Let $x =$ the number of dollars invested at 7% and $y =$ the number of dollars invested at 9%.

	First Investment	Second Investment	Total	
Principal, P	x		\$3700	$\rightarrow x + (\quad) = 3700$
Rate of Interest, r		9%		
Time, t	1 year	1 year		
Interest, I	$0.07x$		\$297	$\rightarrow 0.07x + (\quad) = 297$

Answer

3. \$1800 at 7%; \$1900 at 9%

First Investment	Second Investment	Total	
x	y	\$3700	$\rightarrow x + y = 3700$
7%	9%		
1 year	1 year		
$0.07x$	$0.09y$	\$297	$\rightarrow 0.07x + 0.09y = 297$

2. **Translate.** The total of the amounts of the loans is found in the first row of the table. This gives us one equation:

$$x + y = 16{,}200.$$

Look at the last row of the table. The interest totals \$715. This gives us a second equation:

$$5\%x + 4\%y = 715, \quad \text{or} \quad 0.05x + 0.04y = 715.$$

After we multiply on both sides to clear the decimals, we have

$$5x + 4y = 71{,}500.$$

3. **Solve.** Using either elimination or substitution, we solve the resulting system:

$$x + y = 16{,}200,$$
$$5x + 4y = 71{,}500.$$

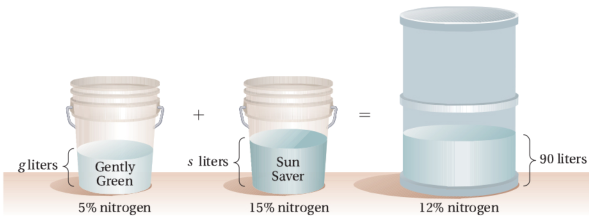
We find that $x = 6700$ and $y = 9500$.

4. **Check.** The sum is \$6700 + \$9500, or \$16,200. The interest from \$6700 at 5% for one year is 5%(\$6700), or \$335. The interest from \$9500 at 4% for one year is 4%(\$9500), or \$380. The total interest is \$335 + \$380, or \$715. The numbers check in the problem.

5. **State.** The Perkins loan was for \$6700 and the Stafford loan was for \$9500.

Do Exercise 3.

EXAMPLE 4 ***Mixing Fertilizers.*** Yardbird Gardening carries two kinds of fertilizer containing nitrogen and water. "Gently Green" is 5% nitrogen and "Sun Saver" is 15% nitrogen. Yardbird Gardening needs to combine the two types of solution to make 90 L of a solution that is 12% nitrogen. How much of each brand should be used?



1. **Familiarize.** We first make a drawing and a guess to become familiar with the problem.

We choose two numbers that total 90 L—say, 40 L of Gently Green and 50 L of Sun Saver—for the amounts of each fertilizer. Will the resulting mixture have the correct percentage of nitrogen?

To find out, we multiply as follows:

$$5\%(40\text{ L}) = 2\text{ L of nitrogen} \quad \text{and} \quad 15\%(50\text{ L}) = 7.5\text{ L of nitrogen}.$$

Thus the total amount of nitrogen in the mixture is $2\text{ L} + 7.5\text{ L}$, or 9.5 L. The final mixture of 90 L is supposed to be 12% nitrogen. Now

$$12\%(90\text{ L}) = 10.8\text{ L}.$$

Since 9.5 L and 10.8 L are not the same, our guess is incorrect. But these calculations help us to become familiar with the problem and to make the translation.

We let $g =$ the number of liters of Gently Green and $s =$ the number of liters of Sun Saver in the mixture.

The information can be organized in a table, as follows.

	GENTLY GREEN	SUN SAVER	MIXTURE	
NUMBER OF LITERS	g	s	90	$\rightarrow g + s = 90$
PERCENT OF NITROGEN	5%	15%	12%	
AMOUNT OF NITROGEN	$0.05g$	$0.15s$	0.12×90, or 10.8 liters	$\rightarrow 0.05g + 0.15s = 10.8$

2. **Translate.** If we add g and s in the first row, we get 90, and this gives us one equation:

$$g + s = 90.$$

If we add the amounts of nitrogen listed in the third row, we get 10.8, and this gives us another equation:

$$5\%g + 15\%s = 10.8, \quad \text{or} \quad 0.05g + 0.15s = 10.8.$$

After clearing the decimals, we have the following system:

$$g + s = 90, \qquad \textbf{(1)}$$
$$5g + 15s = 1080. \qquad \textbf{(2)}$$

3. **Solve.** We solve the system using elimination. We multiply equation (1) by -5 and add the result to equation (2):

$$\begin{aligned} -5g - 5s &= -450 && \text{Multiplying equation (1) by } -5 \\ 5g + 15s &= 1080 && \text{Equation (2)} \\ \hline 10s &= 630 && \text{Adding} \\ s &= 63. && \text{Dividing by 10} \end{aligned}$$

Next, we substitute 63 for s in equation (1) and solve for g:

$$\begin{aligned} g + 63 &= 90 && \text{Substituting in equation (1)} \\ g &= 27. && \text{Solving for } g \end{aligned}$$

We obtain $(27, 63)$, or $g = 27$, $s = 63$.

4. **Mixing Cleaning Solutions.** King's Service Station uses two kinds of cleaning solution containing acid and water. "Attack" is 2% acid and "Blast" is 6% acid. They want to mix the two to get 60 qt of a solution that is 5% acid. How many quarts of each should they use?

Do the *Familiarize* and *Translate* steps by completing the following table. Let a = the number of quarts of Attack and b = the number of quarts of Blast.

	ATTACK	BLAST	MIXTURE	
AMOUNT OF SOLUTION	a	b		$\rightarrow a + b = (\quad)$
PERCENT OF ACID	2%			
AMOUNT OF ACID IN SOLUTION		$0.06b$		$\rightarrow (\quad) + 0.06b = (\quad)$

4. **Check.** Remember that g is the number of liters of Gently Green, with 5% nitrogen, and s is the number of liters of Sun Saver, with 15% nitrogen.

Total number of liters of mixture: $g + s = 27 + 63 = 90$ L

Amount of nitrogen: $5\%(27) + 15\%(63) = 1.35 + 9.45 = 10.8$ L

Percentage of nitrogen in mixture: $\dfrac{10.8}{90} = 0.12 = 12\%$

The numbers check in the original problem.

5. **State.** Yardbird Gardening should mix 27 L of Gently Green and 63 L of Sun Saver.

Do Exercise 4.

b Motion Problems

When a problem deals with speed, distance, and time, we can expect to use the following *motion formula*.

THE MOTION FORMULA

Distance = Rate (or speed) · Time

$d = rt$

TIPS FOR SOLVING MOTION PROBLEMS

1. Make a drawing using an arrow or arrows to represent distance and the direction of each object in motion.
2. Organize the information in a table or a chart.
3. Look for as many things as you can that are the same, so you can write equations.

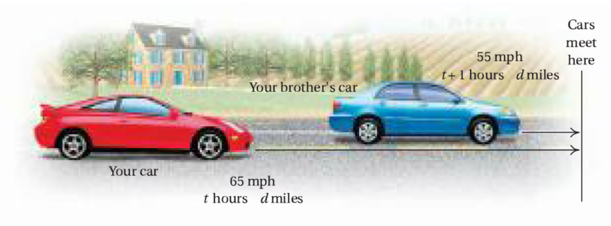
Answer

4. Attack: 15 qt; Blast: 45 qt

Attack	Blast	Mixture	
a	b	60	$\rightarrow a + b = 60$
2%	6%	5%	
$0.02a$	$0.06b$	0.05×60, or 3	$\rightarrow 0.02a + 0.06b = 3$

EXAMPLE 5 *Auto Travel.* Your brother leaves your home on a trip, forgetting his suitcase. You know that he normally drives at a speed of 55 mph. You do not discover the suitcase until 1 hr after he has left. If you follow him at a speed of 65 mph, how long will it take you to catch up with him?

1. **Familiarize.** We first make a drawing. From the drawing, we see that when you catch up with your brother, the distances from home are the same. We let d = the distance, in miles. If we let t = the time, in hours, for you to catch your brother, then $t + 1$ = the time traveled by your brother at a slower speed.



We organize the information in a table as follows.

	DISTANCE (d)	RATE (r)	TIME (t)	
BROTHER	d	55	$t + 1$	→ $d = 55(t + 1)$
YOU	d	65	t	→ $d = 65t$

2. **Translate.** Using $d = rt$ in each row of the table, we get an equation. Thus we have a system of equations:

$$d = 55(t + 1), \quad \textbf{(1)}$$
$$d = 65t. \quad \textbf{(2)}$$

3. **Solve.** We solve the system using the substitution method:

$65t = 55(t + 1)$ Substituting $65t$ for d in equation (1)

$65t = 55t + 55$ Multiplying to remove parentheses on the right

$\left.\begin{aligned} 10t &= 55 \\ t &= 5.5. \end{aligned}\right\}$ Solving for t

Your time is 5.5 hr, which means that your brother's time is 5.5 + 1, or 6.5 hr.

4. **Check.** At 65 mph, you will travel $65 \cdot 5.5$, or 357.5 mi, in 5.5 hr. At 55 mph, your brother will travel $55 \cdot 6.5$, or the same 357.5 mi, in 6.5 hr. The numbers check.

5. **State.** You will catch up with your brother in 5.5 hr.

Do Exercise 5.

5. **Train Travel.** A train leaves Barstow traveling east at 35 km/h. One hour later, a faster train leaves Barstow, also traveling east on a parallel track at 40 km/h. How far from Barstow will the faster train catch up with the slower one?

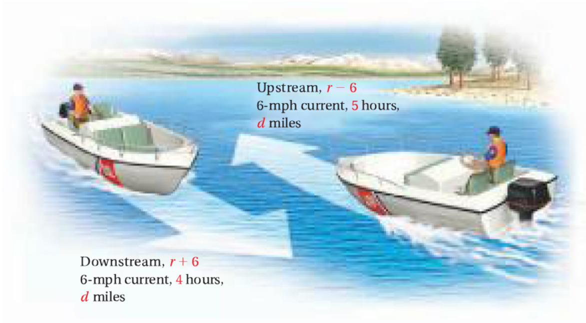
	DISTANCE (d)	RATE (r)	TIME (t)	
SLOWER TRAIN			t	→ $d =$
FASTER TRAIN	d			→ $d =$

Answer

5. 280 km

Distance	Rate	Time	
d	35 km/h	t	→ $d = 35t$
d	40 km/h	$t - 1$	→ $d = 40(t - 1)$

EXAMPLE 6 ***Marine Travel.*** A Coast-Guard patrol boat travels 4 hr on a trip downstream with a 6-mph current. The return trip against the same current takes 5 hr. Find the speed of the boat in still water.



1. **Familiarize.** We first make a drawing. From the drawing, we see that the distances are the same. We let d = the distance, in miles, and r = the speed of the boat in still water, in miles per hour. Then, when the boat is traveling downstream, its speed is $r + 6$. (The current helps the boat along.) When it is traveling upstream, its speed is $r - 6$. (The current holds the boat back.) We can organize the information in a table. We use the formula $d = rt$.

$$d = r \cdot t$$

	DISTANCE	RATE	TIME	
DOWNSTREAM	d	$r + 6$	4	$\rightarrow d = (r + 6)4$
UPSTREAM	d	$r - 6$	5	$\rightarrow d = (r - 6)5$

2. **Translate.** From each row of the table, we get an equation, $d = rt$:

$$d = 4r + 24, \quad \textbf{(1)}$$
$$d = 5r - 30. \quad \textbf{(2)}$$

3. **Solve.** We solve the system by the substitution method:

$$4r + 24 = 5r - 30 \qquad \text{Substituting } 4r + 24 \text{ for } d \text{ in equation (2)}$$
$$\left.\begin{aligned} 24 &= r - 30 \\ 54 &= r. \end{aligned}\right\} \quad \text{Solving for } r$$

4. **Check.** If $r = 54$, then $r + 6 = 60$; and $60 \cdot 4 = 240$, the distance traveled downstream. If $r = 54$, then $r - 6 = 48$; and $48 \cdot 5 = 240$, the distance traveled upstream. The distances are the same. In this type of problem, a problem-solving tip to keep in mind is "Have I found what the problem asked for?" We could solve for a certain variable but still have not answered the question of the original problem. For example, we might have found speed when the problem wanted distance. In this problem, we want the speed of the boat in still water, and that is r.

5. **State.** The speed in still water is 54 mph.

Do Exercise 6.

6. **Air Travel.** An airplane flew for 4 hr with a 20-mph tailwind. The return flight against the same wind took 5 hr. Find the speed of the plane in still air.

$$d = r \cdot t$$

	DISTANCE	RATE	TIME	
WITH WIND		$r + 20$		$\rightarrow d =$
AGAINST WIND	d			$\rightarrow d =$

Answer

6. 180 mph

	Distance	Rate	Time	
	d	$r + 20$	4 hr	$\rightarrow d = 4(r + 20)$
	d	$r - 20$	5 hr	$\rightarrow d = 5(r - 20)$

Translating for Success

1. ***Office Expense.*** The monthly telephone expense for an office is \$1094 less than the janitorial expense. Three times the janitorial expense minus four times the telephone expense is \$248. What is the total of the two expenses?

2. ***Dimensions of a Triangle.*** The sum of the base and the height of a triangle is 192 in. The height is twice the base. Find the base and the height.

3. ***Supplementary Angles.*** Two supplementary angles are such that twice one angle is 7° more than the other. Find the measures of the angles.

4. ***SAT Scores.*** The total of Megan's writing and math scores on the SAT was 1094. Her math score was 248 points higher than her writing score. What were her math and writing SAT scores?

5. ***Sightseeing Boat.*** A sightseeing boat travels 3 hr on a trip downstream with a 2.5-mph current. The return trip against the same current takes 3.5 hr. Find the speed of the boat in still water.

The goal of these matching questions is to practice step (2), *Translate*, of the five-step problem-solving process. Translate each word problem to a system of equations and select a correct translation from systems A–J.

A. $x = y + 248,$
$x + y = 1094$

B. $5x = 2y - 3,$
$y = \frac{2}{3}x + 5$

C. $y = \frac{1}{2}x,$
$2x + 2y = 192$

D. $2x = 7 + y,$
$x + y = 180$

E. $x + y = 192,$
$x = 2y$

F. $x + y = 180,$
$x = 2y + 7$

G. $x - 1094 = y,$
$3x - 4y = 248$

H. $3\%x + 2.5\%y = 97.50,$
$x + y = 2500$

I. $2x = 5 + \frac{2}{3}y,$
$3y = 15x - 4$

J. $x = (y + 2.5) \cdot 3,$
$3.5(y - 2.5) = x$

Answers on page A-24

6. ***Running Distances.*** Each day Tricia runs 5 mi more than two-thirds the distance that Chris runs. Five times the distance that Chris runs is 3 mi less than twice the distance that Tricia runs. How far does Tricia run daily?

7. ***Dimensions of a Rectangle.*** The perimeter of a rectangle is 192 in. The width is half the length. Find the length and the width.

8. ***Mystery Numbers.*** Teka asked her students to determine the two numbers that she placed in a sealed envelope. Twice the smaller number is 5 more than two-thirds the larger number. Three times the larger number is 4 less than fifteen times the smaller. Find the numbers.

9. ***Supplementary Angles.*** Two supplementary angles are such that one angle is 7° more than twice the other. Find the measures of the angles.

10. ***Student Loans.*** Brandt's student loans totaled \$2500. Part was borrowed at 3% interest and the rest at 2.5%. After one year, Brandt had accumulated \$97.50 in interest. What was the amount of each loan?

8.4 Exercise Set

For Extra Help

MyMathLab

MathXL PRACTICE | WATCH | DOWNLOAD | READ | REVIEW

a Solve.

1. ***Retail Sales.*** Paint Town sold 45 paintbrushes, one kind at \$8.50 each and another at \$9.75 each. In all, \$398.75 was taken in for the brushes. How many of each kind were sold?

2. ***Retail Sales.*** Mountainside Fleece sold 40 neckwarmers. Solid-color neckwarmers sold for \$9.90 each and print ones sold for \$12.75 each. In all, \$421.65 was taken in for the neckwarmers. How many of each type were sold?

3. ***Sales of Pharmaceuticals.*** In 2009, the Diabetic Express charged \$39.95 for a vial of Humulin insulin and \$30.49 for a vial of Novolin insulin. If a total of \$1723.16 was collected for 50 vials of insulin, how many vials of each type were sold?

4. ***Fundraising.*** The St. Mark's Community Barbecue served 250 dinners. A child's plate cost \$3.50 and an adult's plate cost \$7.00. A total of \$1347.50 was collected. How many of each type of plate was served?

5. ***Radio Airplay.*** Rudy must play 12 commercials during his 1-hr radio show. Each commercial is either 30 sec or 60 sec long. If the total commercial time during the hour is 10 min, how many commercials of each type does Rudy play?

6. ***Nontoxic Floor Wax.*** A nontoxic floor wax can be made by combining lemon juice and food-grade linseed oil. The amount of oil should be twice the amount of lemon juice. How much of each ingredient is needed in order to make 32 oz of floor wax? (The mix should be spread with a rag and buffed when dry.)

7. ***Catering.*** Stella's Catering is planning a wedding reception. The bride and groom would like to serve a nut mixture containing 25% peanuts. Stella has available mixtures that are either 40% or 10% peanuts. How much of each type should be mixed to get a 10-lb mixture that is 25% peanuts?

8. ***Blending Granola.*** Deep Thought Granola is 25% nuts and dried fruit. Oat Dream Granola is 10% nuts and dried fruit. How much of Deep Thought and how much of Oat Dream should be mixed to form a 20-lb batch of granola that is 19% nuts and dried fruit?

9. ***Ink Remover.*** Etch Clean Graphics uses one cleanser that is 25% acid and a second that is 50% acid. How many liters of each should be mixed to get 10 L of a solution that is 40% acid?

10. ***Livestock Feed.*** Soybean meal is 16% protein and corn meal is 9% protein. How many pounds of each should be mixed to get a 350-lb mixture that is 12% protein?

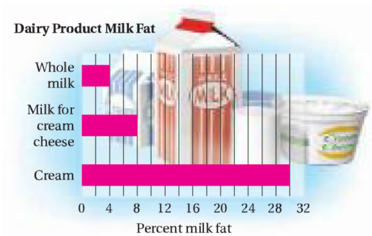
11. ***Dry Cleaners.*** Claudio, a banking vice-president, took 17 neckties to Milto Cleaners. The rate for non-silk ties is \$3.25 per tie and for silk ties is \$3.60 per tie. His total bill was \$58.75. How many silk ties did he have dry-cleaned?

12. ***Laundry.*** While on a four-week hiking trip in the mountains, the Tryon family washed 11 loads of clothes at The Mountain View Laundry. The 20-lb capacity washing machine costs \$1.50 per load while the 30-lb costs \$2.50. Their total laundry expense was \$20.50. How many loads were laundered in each size washing machine?

13. ***Student Loans.*** Sarah's two student loans totaled \$12,000. One of her loans was at 6% simple interest and the other at 9%. After one year, Sarah owed \$855 in interest. What was the amount of each loan?

14. ***Investments.*** An executive nearing retirement made two investments totaling \$45,000. In one year, these investments yielded \$2430 in simple interest. Part of the money was invested at 4% and the rest at 6%. How much was invested at each rate?

15. ***Food Science.*** The following bar graph shows the milk fat percentages in three dairy products. How many pounds each of whole milk and cream should be mixed in order to form 200 lb of milk for cream cheese?



16. ***Automotive Maintenance.*** Arctic Antifreeze is 18% alcohol and Frost No-More is 10% alcohol. How many liters of Arctic Antifreeze should be mixed with 7.5 L of Frost No-More in order to get a mixture that is 15% alcohol?

17. ***Teller Work.*** Juan goes to a bank and gets change for a \$50 bill consisting of all \$5 bills and \$1 bills. There are 22 bills in all. How many of each kind are there?

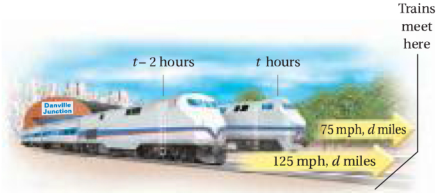
18. ***Making Change.*** Christina makes a \$9.25 purchase at a bookstore in Reno with a \$20 bill. The store has no bills and gives her the change in quarters and dollar coins. There are 19 coins in all. How many of each kind are there?

19. ***Investments.*** William opened two investment accounts for his grandson's college fund. The first year, these investments, which totaled \$18,000, yielded \$831 in simple interest. Part of the money was invested at 5.5% and the rest at 4%. How much was invested at each rate?

20. ***Student Loans.*** Cole's two student loans totaled \$31,000. One of his loans was at 2.8% simple interest and the other at 4.5%. After one year, Cole owed \$1024.40 in interest. What was the amount of each loan?

b Solve.

21. ***Train Travel.*** A train leaves Danville Junction and travels north at a speed of 75 mph. Two hours later, a second train leaves on a parallel track and travels north at 125 mph. How far from the station will they meet?



22. ***Car Travel.*** Two cars leave Denver traveling in opposite directions. One car travels at a speed of 80 km/h and the other at 96 km/h. In how many hours will they be 528 km apart?

23. ***Canoeing.*** Darren paddled for 4 hr with a 6-km/h current to reach a campsite. The return trip against the same current took 10 hr. Find the speed of Darren's canoe in still water.

24. ***Boating.*** Mia's motorboat took 3 hr to make a trip downstream with a 6-mph current. The return trip against the same current took 5 hr. Find the speed of the boat in still water.

25. ***Car Travel.*** Donna is late for a sales meeting after traveling from one town to another at a speed of 32 mph. If she had traveled 4 mph faster, she could have made the trip in $\frac{1}{2}$ hr less time. How far apart are the towns?

26. ***Air Travel.*** Rod is a pilot for Crossland Airways. He computes his flight time against a headwind for a trip of 2900 mi at 5 hr. The flight would take 4 hr and 50 min if the headwind were half as great. Find the headwind and the plane's air speed.

27. ***Air Travel.*** Two planes travel toward each other from cities that are 780 km apart at rates of 190 km/h and 200 km/h. They started at the same time. In how many hours will they meet?

28. ***Motorcycle Travel.*** Sally and Rocky travel on motorcycles toward each other from Chicago and Indianapolis, which are about 350 km apart, and they are biking at rates of 110 km/h and 90 km/h. They started at the same time. In how many hours will they meet?

29. ***Air Travel.*** Two airplanes start at the same time and fly toward each other from points 1000 km apart at rates of 420 km/h and 330 km/h. After how many hours will they meet?

30. ***Truck and Car Travel.*** A truck and a car leave a service station at the same time and travel in the same direction. The truck travels at 55 mph and the car at 40 mph. They can maintain CB radio contact within a range of 10 mi. When will they lose contact?

31. ***Point of No Return.*** A plane flying the 3458-mi trip from New York City to London has a 50-mph tailwind. The flight's *point of no return* is the point at which the flight time required to return to New York is the same as the time required to continue to London. If the speed of the plane in still air is 360 mph, how far is New York from the point of no return?

32. ***Point of No Return.*** A plane is flying the 2553-mi trip from Los Angeles to Honolulu into a 60-mph headwind. If the speed of the plane in still air is 310 mph, how far from Los Angeles is the plane's point of no return? (See Exercise 31.)

Skill Maintenance

Given the function $f(x) = 4x - 7$, find each of the following function values. [7.1b]

33. $f(0)$ **34.** $f(-1)$ **35.** $f(1)$ **36.** $f(10)$

37. $f(-2)$ **38.** $f(2a)$ **39.** $f(-4)$ **40.** $f(1.8)$

41. $f\left(\frac{3}{4}\right)$ **42.** $f(-2.5)$ **43.** $f(-3h)$ **44.** $f(1000)$

Synthesis

45. ***Automotive Maintenance.*** The radiator in Michelle's car contains 16 L of antifreeze and water. This mixture is 30% antifreeze. How much of this mixture should she drain and replace with pure antifreeze so that there will be a mixture of 50% antifreeze?

46. ***Physical Exercise.*** Natalie jogs and walks to school each day. She averages 4 km/h walking and 8 km/h jogging. The distance from home to school is 6 km and Natalie makes the trip in 1 hr. How far does she jog in a trip?

47. ***Fuel Economy.*** Sally Cline's SUV gets 18 miles per gallon (mpg) in city driving and 24 mpg in highway driving. The SUV is driven 465 mi on 23 gal of gasoline. How many miles were driven in the city and how many were driven on the highway?

48. ***Siblings.*** Phil and Phyllis are siblings. Phyllis has twice as many brothers as she has sisters. Phil has the same number of brothers as sisters. How many girls and how many boys are in the family?

49. ***Wood Stains.*** Bennet Custom Flooring has 0.5 gal of stain that is 20% brown and 80% neutral. A customer orders 1.5 gal of a stain that is 60% brown and 40% neutral. How much pure brown stain and how much neutral stain should be added to the original 0.5 gal in order to make up the order?

50. See Exercise 49. Let $x =$ the amount of pure brown stain added to the original 0.5 gal. Find a function $P(x)$ that can be used to determine the percentage of brown stain in the 1.5-gal mixture. On a graphing calculator, draw the graph of P and use ZOOM and TRACE or the TABLE feature to confirm the answer to Exercise 49.

Mid-Chapter Review

Concept Reinforcement

Determine whether each statement is true or false.

______ **1.** If, when solving a system of two linear equations in two variables, a false equation is obtained, the system has infinitely many solutions. [8.2a], [8.3a]

______ **2.** Every system of equations has at least one solution. [8.1a]

______ **3.** If the graphs of two linear equations intersect, then the system is consistent. [8.1a]

______ **4.** The intersection of the graphs of the lines $x = a$ and $y = b$ is (a, b). [8.1a]

Guided Solutions

Fill in each box with the number, variable, or expression that creates a correct statement or solution.

Solve. [8.2a], [8.3a]

5. $x + 2y = 3,$ **(1)**
$y = x - 6$ **(2)**

$x + 2(\square) = 3$ Substituting for y in equation (1)

$x + \square x - \square = 3$ Removing parentheses

$\square x - 12 = 3$ Collecting like terms

$3x = \square$

$x = \square$

$y = \square - 6$ Substituting in equation (2)

$y = \square$ Subtracting

The solution is $(\square, \square)$.

6. $3x - 2y = 5,$ **(1)**
$2x + 4y = 14$ **(2)**

$\square x - \square y = \square$ Multiplying equation (1) by 2

$2x + 4y = 14$ Equation (2)

$\square x = \square$ Adding

$x = \square$

$2 \cdot \square + 4y = 14$ Substituting for x in equation (2)

$\square + 4y = 14$ Multiplying

$4y = \square$

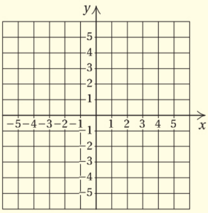
$y = \square$

The solution is $(\square, \square)$.

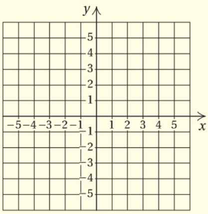
Mixed Review

Solve each system of equations graphically. Then classify the system as consistent or inconsistent and the equations as dependent or independent. [8.1a]

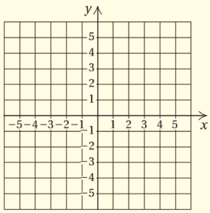
7. $y = x - 6,$
$y = 4 - x$



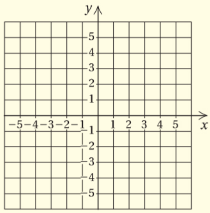
8. $x + y = 3,$
$3x + y = 3$



9. $y = 2x - 3,$
$4x - 2y = 6$



10. $x - y = 3,$
$2y - 2x = 6$



Solve using the substitution method. [8.2a]

11. $x = y + 2,$
$2x - 3y = -2$

12. $y = x - 5,$
$x - 2y = 8$

13. $4x + 3y = 3,$
$y = x + 8$

14. $3x - 2y = 1,$
$x = y + 1$

Solve using the elimination method. [8.3a]

15. $2x + y = 2,$
$x - y = 4$

16. $x - 2y = 13,$
$x + 2y = -3$

17. $3x - 4y = 5,$
$5x - 2y = -1$

18. $3x + 2y = 11,$
$2x + 3y = 9$

19. $x - 2y = 5,$
$3x - 6y = 10$

20. $4x - 6y = 2,$
$-2x + 3y = -1$

21. $\frac{1}{2}x + \frac{1}{3}y = 1,$
$\frac{1}{5}x - \frac{3}{4}y = 11$

22. $0.2x + 0.3y = 0.6,$
$0.1x - 0.2y = -2.5$

Solve.

23. ***Garden Dimensions.*** A landscape architect designs a garden with a perimeter of 44 ft. The width is 2 ft less than the length. Find the length and the width. [8.2b]

24. ***Investments.*** Sandy made two investments totaling \$5000. Part of the money was invested at 2% and the rest at 3%. In one year, these investments earned \$129 in simple interest. How much was invested at each rate? [8.4a]

25. ***Mixing Solutions.*** A lab technician wants to mix a solution that is 20% acid with a second solution that is 50% acid in order to get 84 L of a solution that is 30% acid. How many liters of each solution should be used? [8.4a]

26. ***Boating.*** Monica's motorboat took 5 hr to make a trip downstream with a 6-mph current. The return trip against the same current took 8 hr. Find the speed of the boat in still water. [8.4b]

Understanding Through Discussion and Writing

27. Explain how to find the solution of $\frac{3}{4}x + 2 = \frac{2}{5}x - 5$ in two ways graphically and in two ways algebraically. [8.1a], [8.2a], [8.3a]

28. Write a system of equations with the given solution. Answers may vary. [8.1a], [8.2a], [8.3a]

a) $(4, -3)$
b) No solution
c) Infinitely many solutions

29. Describe a method that could be used to create an inconsistent system of equations. [8.1a], [8.2a], [8.3a]

30. Describe a method that could be used to create a system of equations with dependent equations. [8.1a], [8.2a], [8.3a]

8.5 Systems of Equations in Three Variables

OBJECTIVE

a Solve systems of three equations in three variables.

SKILL TO REVIEW

Objective 8.3a: Solve systems of equations in two variables by the elimination method.

Solve.

1. $3x + y = 1,$
   $5x - y = 7$
2. $2x + 3y = 9,$
   $3x + 2y = 1$

a Solving Systems in Three Variables

A **linear equation in three variables** is an equation equivalent to one of the type $Ax + By + Cz = D$. A **solution** of a system of three equations in three variables is an ordered triple (x, y, z) that makes *all three* equations true.

The substitution method can be used to solve systems of three equations, but it is not efficient unless a variable has already been eliminated from one or more of the equations. Therefore, we will use only the elimination method—essentially the same procedure for systems of three equations as for systems of two equations.* The first step is to eliminate a variable and obtain a system of two equations in two variables.

EXAMPLE 1 Solve the following system of equations:

$$\begin{aligned} x + y + z &= 4, && (1)\\ x - 2y - z &= 1, && (2)\\ 2x - y - 2z &= -1. && (3) \end{aligned}$$

a) We first use *any* two of the three equations to get an equation in two variables. In this case, let's use equations (1) and (2) and add to eliminate z:

$$\begin{aligned} x + y + z &= 4 && (1)\\ x - 2y - z &= 1 && (2)\\ \hline 2x - y \quad &= 5. && (4) \quad \text{Adding to eliminate } z \end{aligned}$$

b) We use a *different* pair of equations and eliminate the **same variable** that we did in part (a). Let's use equations (1) and (3) and again eliminate z.

Caution!

A common error is to eliminate a different variable the second time.

$$\begin{aligned} x + y + z &= 4, && (1)\\ 2x - y - 2z &= -1; && (3) \end{aligned}$$

$$\begin{aligned} 2x + 2y + 2z &= 8 && \text{Multiplying equation (1) by 2}\\ 2x - y - 2z &= -1 && (3)\\ \hline 4x + y \quad &= 7 && (5) \quad \text{Adding to eliminate } z \end{aligned}$$

c) Now we solve the resulting system of equations, (4) and (5). That solution will give us two of the numbers. Note that we now have two equations in two variables. Had we eliminated two *different* variables in parts (a) and (b), this would not be the case.

$$\begin{aligned} 2x - y &= 5 && (4)\\ 4x + y &= 7 && (5)\\ \hline 6x \quad &= 12 && \text{Adding}\\ x &= 2 \end{aligned}$$

STUDY TIPS

WORKED-OUT-SOLUTIONS

The *Student's Solutions Manual* is an excellent resource if you need additional help with an exercise in the exercise sets. It contains step-by-step solutions to each odd-numbered exercise.

Answers

Skill to Review:
1. $(1, -2)$ 2. $(-3, 5)$

*Other methods for solving systems of equations are considered in Appendixes I and J.

We can use either equation (4) or (5) to find y. We choose equation (5):

$$\begin{aligned} 4x + y &= 7 && \textbf{(5)} \\ 4(2) + y &= 7 && \text{Substituting 2 for } x \\ 8 + y &= 7 \\ y &= -1. \end{aligned}$$

d) We now have $x = 2$ and $y = -1$. To find the value for z, we use any of the original three equations and substitute to find the third number, z. Let's use equation (1) and substitute our two numbers in it:

$$\begin{aligned} x + y + z &= 4 && \textbf{(1)} \\ 2 + (-1) + z &= 4 && \text{Substituting 2 for } x \text{ and } -1 \text{ for } y \\ \left.\begin{aligned} 1 + z &= 4 \\ z &= 3. \end{aligned}\right\} & && \text{Solving for } z \end{aligned}$$

We have obtained the ordered triple $(2, -1, 3)$. We check as follows, substituting $(2, -1, 3)$ into each of the three equations using alphabetical order.

Check:

$$\begin{array}{r|l} x + y + z = & 4 \\ \hline 2 + (-1) + 3 \stackrel{?}{=} & 4 \\ 4 & \quad \text{TRUE} \end{array}$$

$$\begin{array}{r|l} x - 2y - z = & 1 \\ \hline 2 - 2(-1) - 3 \stackrel{?}{=} & 1 \\ 2 + 2 - 3 & \\ 1 & \quad \text{TRUE} \end{array}$$

$$\begin{array}{r|l} 2x - y - 2z = & -1 \\ \hline 2(2) - (-1) - 2 \cdot 3 \stackrel{?}{=} & -1 \\ 4 + 1 - 6 & \\ -1 & \quad \text{TRUE} \end{array}$$

The triple $(2, -1, 3)$ checks and is the solution.

To use the elimination method to solve systems of three equations:

1. Write all equations in the standard form, $Ax + By + Cz = D$.
2. Clear any decimals or fractions.
3. Choose a variable to eliminate. Then use *any* two of the three equations to eliminate that variable, getting an equation in two variables.
4. Next, use a different pair of equations and get another equation in *the same two variables*. That is, eliminate the same variable that you did in step (3).
5. Solve the resulting system (pair) of equations. That will give two of the numbers.
6. Then use any of the original three equations to find the third number.

Do Exercise 1.

1. Solve. Don't forget to check.

$$\begin{aligned} 4x - y + z &= 6, \\ -3x + 2y - z &= -3, \\ 2x + y + 2z &= 3 \end{aligned}$$

Answer

1. $(2, 1, -1)$

EXAMPLE 2 Solve this system:

$$\begin{aligned} 4x - 2y - 3z &= 5, & \quad \textbf{(1)}\\ -8x - y + z &= -5, & \quad \textbf{(2)}\\ 2x + y + 2z &= 5. & \quad \textbf{(3)} \end{aligned}$$

a) The equations are in standard form and do not contain decimals or fractions.

b) We decide to eliminate the variable y since the y-terms are opposites in equations (2) and (3). We add:

$$\begin{array}{rl} -8x - y + z = -5 & \quad \textbf{(2)} \\ \underline{2x + y + 2z = 5} & \quad \textbf{(3)} \\ -6x \qquad + 3z = 0. & \quad \textbf{(4)} \quad \text{Adding} \end{array}$$

c) We use another pair of equations to get an equation in the same two variables, x and z. That is, we eliminate the same variable, y, that we did in step (b). We use equations (1) and (3) and eliminate y:

$$\begin{aligned} 4x - 2y - 3z &= 5, & \quad \textbf{(1)}\\ 2x + y + 2z &= 5; & \quad \textbf{(3)} \end{aligned}$$

$$\begin{array}{rl} 4x - 2y - 3z = 5 & \quad \textbf{(1)} \\ \underline{4x + 2y + 4z = 10} & \quad \text{Multiplying equation (3) by 2} \\ 8x \qquad + z = 15. & \quad \textbf{(5)} \quad \text{Adding} \end{array}$$

d) Now we solve the resulting system of equations (4) and (5). That will give us two of the numbers:

$$\begin{aligned} -6x + 3z &= 0, & \quad \textbf{(4)}\\ 8x + z &= 15. & \quad \textbf{(5)} \end{aligned}$$

We multiply equation (5) by -3. (We could also have multiplied equation (4) by $-\frac{1}{3}$.)

$$\begin{array}{rl} -6x + 3z = 0 & \quad \textbf{(4)} \\ \underline{-24x - 3z = -45} & \quad \text{Multiplying equation (5) by } -3 \\ -30x \qquad = -45 & \quad \text{Adding} \\ x = \frac{-45}{-30} = \frac{3}{2} & \end{array}$$

We now use equation (5) to find z:

$$\begin{aligned} 8x + z &= 15 & \quad \textbf{(5)}\\ 8\left(\tfrac{3}{2}\right) + z &= 15 & \quad \text{Substituting } \tfrac{3}{2} \text{ for } x\\ 12 + z &= 15 & \\ z &= 3. & \quad \text{Solving for } z \end{aligned}$$

e) Next, we use any of the original equations and substitute to find the third number, y. We choose equation (3) since the coefficient of y there is 1:

$$\begin{aligned} 2x + y + 2z &= 5 & \quad \textbf{(3)}\\ 2\left(\tfrac{3}{2}\right) + y + 2(3) &= 5 & \quad \text{Substituting } \tfrac{3}{2} \text{ for } x \text{ and 3 for } z\\ 3 + y + 6 &= 5 & \\ y + 9 &= 5 & \quad \text{Solving for } y\\ y &= -4. & \end{aligned}$$

The solution is $\left(\frac{3}{2}, -4, 3\right)$. The check is as follows.

Check:

$$\begin{array}{r|l} 4x - 2y - 3z & = 5 \\ \hline 4 \cdot \frac{3}{2} - 2(-4) - 3(3) & \stackrel{?}{=} 5 \\ 6 + 8 - 9 & \\ 5 & \quad \text{TRUE} \end{array}$$

$$\begin{array}{r|l} -8x - y + z & = -5 \\ \hline -8 \cdot \frac{3}{2} - (-4) + 3 & \stackrel{?}{=} -5 \\ -12 + 4 + 3 & \\ -5 & \quad \text{TRUE} \end{array}$$

$$\begin{array}{r|l} 2x + y + 2z & = 5 \\ \hline 2 \cdot \frac{3}{2} + (-4) + 2(3) & \stackrel{?}{=} 5 \\ 3 - 4 + 6 & \\ 5 & \quad \text{TRUE} \end{array}$$

Do Exercise 2.

2. Solve. Don't forget to check.

$$\begin{aligned} 2x + y - 4z &= 0, \\ x - y + 2z &= 5, \\ 3x + 2y + 2z &= 3 \end{aligned}$$

In Example 3, two of the equations have a missing variable.

EXAMPLE 3 Solve this system:

$$\begin{aligned} x + y + z &= 180, && (1) \\ x - z &= -70, && (2) \\ 2y - z &= 0. && (3) \end{aligned}$$

We note that there is no y in equation (2). In order to have a system of two equations in the variables x and z, we need to find another equation without a y. We use equations (1) and (3) to eliminate y:

$$\begin{aligned} x + y + z &= 180, && (1) \\ 2y - z &= 0; && (3) \end{aligned}$$

$$\begin{array}{rrrrl} -2x & - 2y & - 2z = & -360 & \text{Multiplying equation (1) by } -2 \\ & 2y & - z = & 0 & (3) \\ \hline -2x & & - 3z = & -360. & (4) \quad \text{Adding} \end{array}$$

Now we solve the resulting system of equations (2) and (4):

$$\begin{aligned} x - z &= -70, && (2) \\ -2x - 3z &= -360; && (4) \end{aligned}$$

$$\begin{array}{rl} 2x - 2z = -140 & \text{Multiplying equation (2) by 2} \\ -2x - 3z = -360 & (4) \\ \hline -5z = -500 & \text{Adding} \\ z = 100. & \end{array}$$

To find x, we substitute 100 for z in equation (2) and solve for x:

$$\begin{aligned} x - z &= -70 \\ x - 100 &= -70 \\ x &= 30. \end{aligned}$$

Answer

2. $\left(2, -2, \frac{1}{2}\right)$

To find y, we substitute 100 for z in equation (3) and solve for y:

$$\begin{aligned} 2y - z &= 0 \\ 2y - 100 &= 0 \\ 2y &= 100 \\ y &= 50. \end{aligned}$$

The triple $(30, 50, 100)$ is the solution. The check is left to the student.

3. Solve. Don't forget to check.

$$\begin{aligned} x + y + z &= 100, \\ x - y \quad\;\; &= -10, \\ x \quad\;\; - z &= -30 \end{aligned}$$

Do Exercise 3.

It is possible for a system of three equations to have no solution, that is, to be inconsistent. An example is the system

$$\begin{aligned} x + \;\; y + \;\; z &= 14, \\ x + \;\; y + \;\; z &= 11, \\ 2x - 3y + 4z &= -3. \end{aligned}$$

Note the first two equations. It is not possible for a sum of three numbers to be both 14 and 11. Thus the system has no solution. We will not consider such systems here, nor will we consider systems with infinitely many solutions, which also exist.

STUDY TIPS

LEARNING RESOURCES

Please see the preface for more information on these resources and others. To order any of our products, call (800) 824-7799 in the United States or (201) 767-5021 outside the United States, or visit your campus bookstore.

- The *Student's Solutions Manual* contains fully worked-out solutions to the odd-numbered exercises in the exercise sets, as well as solutions to all exercises in the Mid-Chapter Reviews, end-of-chapter Review Exercises, Chapter Tests, and Cumulative Reviews. (ISBN: 978-0-321-61362-2)
- *Worksheets for Classroom or Lab Practice* provide a list of learning objectives, vocabulary and practice problems, and extra practice problems with ample work space. (ISBN: 978-0-321-61368-4)
- As described on p. 56 and in the Preface, Video Resources on DVD Featuring Chapter Test Prep Videos provide section-level lectures for every objective and step-by-step solutions to all the Chapter Test exercises in this textbook. The Chapter Test videos are also available on YouTube (search using BittingerIntroInter) and in MyMathLab.
- InterAct Math Tutorial Website (www.interactmath.com) provides algorithmically generated practice exercises that correlate directly to the exercises in the textbook.
- MathXL® Tutorials on CD provide practice exercises correlated at the objective level to the exercises in the textbook. Every practice exercise is accompanied by an example and a guided solution, and selected exercises may also include a video clip to help illustrate a concept.

Answer

3. $(20, 30, 50)$

8.5 Exercise Set

For Extra Help
MyMathLab
MathXL PRACTICE · WATCH · DOWNLOAD · READ · REVIEW

a Solve.

1. $$\begin{aligned} x + y + z &= 2,\\ 2x - y + 5z &= -5,\\ -x + 2y + 2z &= 1 \end{aligned}$$

2. $$\begin{aligned} 2x - y - 4z &= -12,\\ 2x + y + z &= 1,\\ x + 2y + 4z &= 10 \end{aligned}$$

3. $$\begin{aligned} 2x - y + z &= 5,\\ 6x + 3y - 2z &= 10,\\ x - 2y + 3z &= 5 \end{aligned}$$

4. $$\begin{aligned} x - y + z &= 4,\\ 3x + 2y + 3z &= 7,\\ 2x + 9y + 6z &= 5 \end{aligned}$$

5. $$\begin{aligned} 2x - 3y + z &= 5,\\ x + 3y + 8z &= 22,\\ 3x - y + 2z &= 12 \end{aligned}$$

6. $$\begin{aligned} 6x - 4y + 5z &= 31,\\ 5x + 2y + 2z &= 13,\\ x + y + z &= 2 \end{aligned}$$

7. $$\begin{aligned} 3a - 2b + 7c &= 13,\\ a + 8b - 6c &= -47,\\ 7a - 9b - 9c &= -3 \end{aligned}$$

8. $$\begin{aligned} x + y + z &= 0,\\ 2x + 3y + 2z &= -3,\\ -x + 2y - 3z &= -1 \end{aligned}$$

9. $$\begin{aligned} 2x + 3y + z &= 17,\\ x - 3y + 2z &= -8,\\ 5x - 2y + 3z &= 5 \end{aligned}$$

10. $$\begin{aligned} 2x + y - 3z &= -4,\\ 4x - 2y + z &= 9,\\ 3x + 5y - 2z &= 5 \end{aligned}$$

11. $$\begin{aligned} 2x + y + z &= -2,\\ 2x - y + 3z &= 6,\\ 3x - 5y + 4z &= 7 \end{aligned}$$

12. $$\begin{aligned} 2x + y + 2z &= 11,\\ 3x + 2y + 2z &= 8,\\ x + 4y + 3z &= 0 \end{aligned}$$

13. $$\begin{aligned} x - y + z &= 4,\\ 5x + 2y - 3z &= 2,\\ 3x - 7y + 4z &= 8 \end{aligned}$$

14. $$\begin{aligned} 2x + y + 2z &= 3,\\ x + 6y + 3z &= 4,\\ 3x - 2y + z &= 0 \end{aligned}$$

15. $$\begin{aligned} 4x - y - z &= 4,\\ 2x + y + z &= -1,\\ 6x - 3y - 2z &= 3 \end{aligned}$$

16. $\begin{aligned} 2r + s + t &= 6, \\ 3r - 2s - 5t &= 7, \\ r + s - 3t &= -10 \end{aligned}$

17. $\begin{aligned} a - 2b - 5c &= -3, \\ 3a + b - 2c &= -1, \\ 2a + 3b + c &= 4 \end{aligned}$

18. $\begin{aligned} x + 4y - z &= 5, \\ 2x - y + 3z &= -5, \\ 4x + 3y + z &= 5 \end{aligned}$

19. $\begin{aligned} 2r + 3s + 12t &= 4, \\ 4r - 6s + 6t &= 1, \\ r + s + t &= 1 \end{aligned}$

20. $\begin{aligned} 10x + 6y + z &= 7, \\ 5x - 9y - 2z &= 3, \\ 15x - 12y + 2z &= -5 \end{aligned}$

21. $\begin{aligned} a + 2b + c &= 1, \\ 7a + 3b - c &= -2, \\ a + 5b + 3c &= 2 \end{aligned}$

22. $\begin{aligned} 3p + 2r &= 11, \\ q - 7r &= 4, \\ p - 6q &= 1 \end{aligned}$

23. $\begin{aligned} x + y + z &= 57, \\ -2x + y &= 3, \\ x - z &= 6 \end{aligned}$

24. $\begin{aligned} 4a + 9b &= 8, \\ 8a + 6c &= -1, \\ 6b + 6c &= -1 \end{aligned}$

25. $\begin{aligned} r + s &= 5, \\ 3s + 2t &= -1, \\ 4r + t &= 14 \end{aligned}$

26. $\begin{aligned} a - 5c &= 17, \\ b + 2c &= -1, \\ 4a - b - 3c &= 12 \end{aligned}$

27. $\begin{aligned} x + y + z &= 105, \\ 10y - z &= 11, \\ 2x - 3y &= 7 \end{aligned}$

Skill Maintenance

Solve for the indicated letter. [2.4b]

28. $F = 3ab$, for a

29. $Q = 4(a + b)$, for a

30. $F = \frac{1}{2}t(c - d)$, for d

31. $F = \frac{1}{2}t(c - d)$, for c

32. $Ax + By = c$, for y

33. $Ax - By = c$, for y

Find the slope and the y-intercept. [7.3b]

34. $y = -\frac{2}{3}x - \frac{5}{4}$

35. $y = 5 - 4x$

36. $2x - 5y = 10$

37. $7x - 6.4y = 20$

Synthesis

Solve.

38. $\begin{aligned} w + x - y + z &= 0, \\ w - 2x - 2y - z &= -5, \\ w - 3x - y + z &= 4, \\ 2w - x - y + 3z &= 7 \end{aligned}$

39. $\begin{aligned} w + x + y + z &= 2, \\ w + 2x + 2y + 4z &= 1, \\ w - x + y + z &= 6, \\ w - 3x - y + z &= 2 \end{aligned}$

8.6 Solving Applied Problems: Three Equations

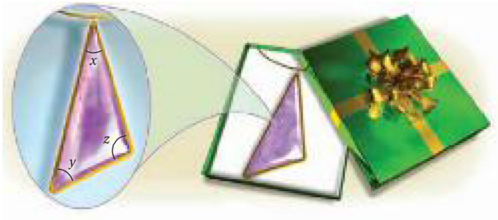
a Using Systems of Three Equations

OBJECTIVE

a Solve applied problems using systems of three equations.

Solving systems of three or more equations is important in many applications occurring in the natural and social sciences, business, and engineering.

EXAMPLE 1 *Jewelry Design.* Kim is designing a triangular-shaped pendant for a client of her custom jewelry business. The largest angle of the triangle is 70° greater than the smallest angle. The largest angle is twice as large as the remaining angle. Find the measure of each angle.



1. **Familiarize.** We first make a drawing. Since we do not know the size of any angle, we use x, y, and z for the measures of the angles. We let $x =$ the smallest angle, $z =$ the largest angle, and $y =$ the remaining angle.
2. **Translate.** In order to translate the problem, we need to make use of a geometric fact—that is, the sum of the measures of the angles of a triangle is 180°. This fact about triangles gives us one equation:

   $$x + y + z = 180.$$

   There are two statements in the problem that we can translate directly.

   $$\underbrace{\text{The largest angle}}_{z} \quad \underbrace{\text{is}}_{=} \quad \underbrace{70°}_{70} \quad \underbrace{\text{greater than}}_{+} \quad \underbrace{\text{the smallest angle.}}_{x}$$

   $$\underbrace{\text{The largest angle}}_{z} \quad \underbrace{\text{is}}_{=} \quad \underbrace{\text{twice as large as the remaining angle.}}_{2y}$$

   We now have a system of three equations:

   $$\begin{aligned} x + y + z &= 180, \\ x + 70 &= z, \\ 2y &= z; \end{aligned} \qquad \text{or} \qquad \begin{aligned} x + y + z &= 180, \\ x \qquad - z &= -70, \\ 2y - z &= 0. \end{aligned}$$

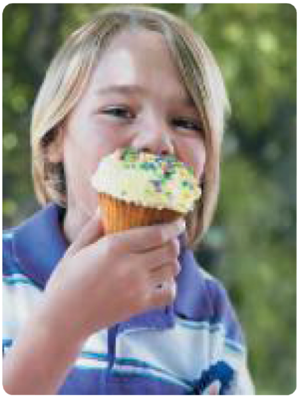
3. **Solve.** The system was solved in Example 3 of Section 8.5. The solution is (30, 50, 100).
4. **Check.** The sum of the numbers is 180. The largest angle measures 100° and the smallest measures 30°, so the largest angle is 70° greater than the smallest. The largest angle is twice as large as 50°, the remaining angle. We have an answer to the problem.
5. **State.** The measures of the angles of the triangle are 30°, 50°, and 100°.

Do Exercise 1.

1. **Triangle Measures.** One angle of a triangle is twice as large as a second angle. The remaining angle is 20° greater than the first angle. Find the measure of each angle.

Answer

1. 64°, 32°, 84°



EXAMPLE 2 *Cholesterol Levels.* Americans have become very conscious of their cholesterol levels. Recent studies indicate that a child's intake of cholesterol should be no more than 300 mg per day. By eating 1 egg, 1 cupcake, and 1 slice of pizza, a child consumes 302 mg of cholesterol. If the child eats 2 cupcakes and 3 slices of pizza, he or she takes in 65 mg of cholesterol. By eating 2 eggs and 1 cupcake, a child consumes 567 mg of cholesterol. How much cholesterol is in each item?

1. **Familiarize.** After we have read the problem a few times, it becomes clear that an egg contains considerably more cholesterol than the other foods. Let's guess that one egg contains 200 mg of cholesterol and one cupcake contains 50 mg. Because of the third sentence in the problem, it would follow that a slice of pizza contains 52 mg of cholesterol since $200 + 50 + 52 = 302$.

   To see if our guess satisfies the other statements in the problem, we find the amount of cholesterol that 2 cupcakes and 3 slices of pizza would contain: $2 \cdot 50 + 3 \cdot 52 = 256$. Since this does not match the 65 mg listed in the fourth sentence of the problem, our guess was incorrect. Rather than guess again, we examine how we checked our guess and let g, c, and $s =$ the number of milligrams of cholesterol in an egg, a cupcake, and a slice of pizza, respectively.

2. **Translate.** By rewording some of the sentences in the problem, we can translate it into three equations.

$$\underbrace{\text{The amount of cholesterol in 1 egg}}_{\downarrow\ g} \quad \underbrace{\text{plus}}_{\downarrow\ +} \quad \underbrace{\text{the amount of cholesterol in 1 cupcake}}_{\downarrow\ c} \quad \underbrace{\text{plus}}_{\downarrow\ +} \quad \underbrace{\text{the amount of cholesterol in 1 slice of pizza}}_{\downarrow\ s} \quad \underbrace{\text{is}}_{=} \quad \underbrace{\text{302 mg.}}_{\downarrow\ 302}$$

$$\underbrace{\text{The amount of cholesterol in 2 cupcakes}}_{\downarrow\ 2c} \quad \underbrace{\text{plus}}_{\downarrow\ +} \quad \underbrace{\text{the amount of cholesterol in 3 slices of pizza}}_{\downarrow\ 3s} \quad \underbrace{\text{is}}_{\downarrow\ =} \quad \underbrace{\text{65 mg.}}_{\downarrow\ 65}$$

$$\underbrace{\text{The amount of cholesterol in 2 eggs}}_{\downarrow\ 2g} \quad \underbrace{\text{plus}}_{\downarrow\ +} \quad \underbrace{\text{the amount of cholesterol in 1 cupcake}}_{\downarrow\ c} \quad \underbrace{\text{is}}_{\downarrow\ =} \quad \underbrace{\text{567 mg.}}_{\downarrow\ 567}$$

   We now have a system of three equations:

$$\begin{aligned} g + c + s &= 302, & \textbf{(1)}\\ 2c + 3s &= 65, & \textbf{(2)}\\ 2g + c &= 567. & \textbf{(3)} \end{aligned}$$

3. **Solve.** To solve, we first note that the variable g does not appear in equation (2). In order to have a system of two equations in the variables c and s, we need to find another equation without the variable g. We use equations (1) and (3) to eliminate g:

$$g + c + s = 302, \quad (1)$$
$$2g + c = 567; \quad (3)$$

$$\begin{array}{rcll} -2g - 2c - 2s &=& -604 & \text{Multiplying equation (1) by } -2 \\ 2g + c &=& 567 & (3) \\ \hline -c - 2s &=& -37. & (4) \quad \text{Adding} \end{array}$$

Next, we solve the resulting system of equations (2) and (4):

$$2c + 3s = 65, \quad (2)$$
$$-c - 2s = -37; \quad (4)$$

$$\begin{array}{rcll} 2c + 3s &=& 65 & (2) \\ -2c - 4s &=& -74 & \text{Multiplying equation (4) by 2} \\ \hline -s &=& -9 & \text{Adding} \\ s &=& 9. & \end{array}$$

To find c, we substitute 9 for s in equation (4) and solve for c:

$$\begin{array}{rcll} -c - 2s &=& -37 & (4) \\ -c - 2(9) &=& -37 & \text{Substituting} \\ -c - 18 &=& -37 & \\ -c &=& -19 & \\ c &=& 19. & \end{array}$$

To find g, we substitute 19 for c in equation (3) and solve for g:

$$\begin{array}{rcll} 2g + c &=& 567 & (3) \\ 2g + 19 &=& 567 & \text{Substituting} \\ 2g &=& 548 & \\ g &=& 274. & \end{array}$$

The solution is $c = 19$, $g = 274$, $s = 9$, or $(19, 274, 9)$.

4. **Check.** The sum of 19, 274, and 9 is 302 so the total cholesterol in 1 cupcake, 1 egg, and 1 slice of pizza checks. Two cupcakes and three slices of pizza would contain $2 \cdot 19 + 3 \cdot 9$, or 65 mg, while two eggs and one cupcake would contain $2 \cdot 274 + 19$, or 567 mg of cholesterol. The answer checks.

5. **State.** A cupcake contains 19 mg of cholesterol, an egg contains 274 mg of cholesterol, and a slice of pizza contains 9 mg of cholesterol.

Do Exercise 2.

2. **Client Investments.** Kaufman Financial Corporation makes investments for corporate clients. One year, a client receives \$1620 in simple interest from three investments that total \$25,000. Part is invested at 5%, part at 6%, and part at 7%. There is \$11,000 more invested at 7% than at 6%. How much was invested at each rate?

Answer

2. \$4000 at 5%; \$5000 at 6%; \$16,000 at 7%

8.6 Exercise Set

For Extra Help

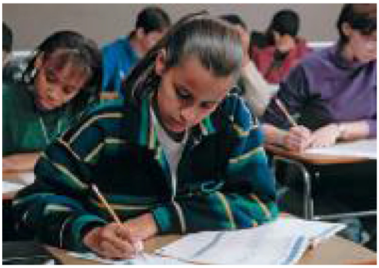
MyMathLab

MathXL PRACTICE | WATCH | DOWNLOAD | READ | REVIEW

a Solve.

1. ***Scholastic Aptitude Test.*** More than two million high-school students take the Scholastic Aptitude Test each year as part of the college admission process. Students receive a critical reading score, a mathematics score, and a writing score. The average total score of students who graduated from high school in 2008 was 1511. The average math score exceeded the average reading score by 13 points. The average math score was 481 points less than the sum of the average reading and writing scores. Find the average score on each part of the test.

   Source: College Board



2. ***Fat Content of Fast Food.*** A meal at McDonald's consisting of a Big Mac, a medium order of fries, and a 21-oz vanilla milkshake contains 66 g of fat. The Big Mac has 11 more grams of fat than the milkshake. The total fat content of the fries and the shake exceeds that of the Big Mac by 8 g. Find the fat content of each food item.

   Source: McDonald's

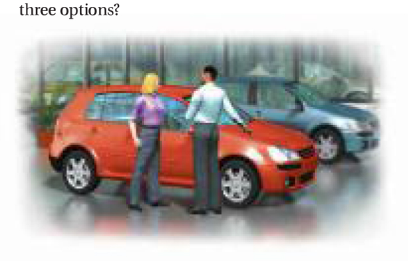
3. ***Triangle Measures.*** In triangle ABC, the measure of angle B is three times that of angle A. The measure of angle C is 20° more than that of angle A. Find the measure of each angle.

4. ***Triangle Measures.*** In triangle ABC, the measure of angle B is twice the measure of angle A. The measure of angle C is 80° more than that of angle A. Find the measure of each angle.

5. The sum of three numbers is 55. The difference of the largest and the smallest is 49, and the sum of the two smaller is 13. Find the numbers.

6. The sum of three numbers is -30. The largest minus twice the smallest is 45, and the largest is 20 more than the middle number. Find the numbers.

7. *Automobile Pricing.* A recent basic model of a particular automobile had a price of \$12,685. The basic model with the added features of automatic transmission and power door locks was \$14,070. The basic model with air conditioning (AC) and power door locks was \$13,580. The basic model with AC and automatic transmission was \$13,925. What was the individual cost of each of the three options?



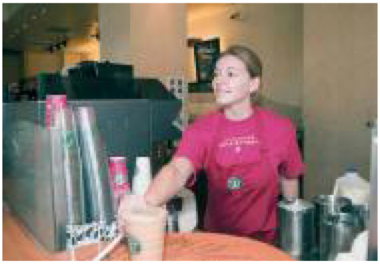
8. *Telemarketing.* Steve, Teri, and Isaiah can process 740 telephone orders per day. Steve and Teri together can process 470 orders, while Teri and Isaiah together can process 520 orders per day. How many orders can each person process alone?

9. *Low-Fat Fruit Drinks.* A Smoothie King® on a large college campus recently sold small low-fat fruit Smoothies for \$4.30 each, medium Smoothies for \$6.50 each, and large Smoothies for \$8.00 each. One hot summer afternoon, Jake sold 34 Smoothies for a total of \$211. The number of small and large Smoothies, combined, was 2 more than the number of medium Smoothies. How many of each size were sold?

Source: campusfood.com

10. *Cappuccinos.* A Starbucks® on campus sells cappuccinos in three sizes: tall for \$2.65, grande for \$3.20, and venti® for \$3.50. One morning, Brianna served 50 cappuccinos. The number of tall and venti® cappuccinos, combined, was 2 fewer than the number of grande cappuccinos. If she collected a total of \$157, how many cappuccinos of each size were sold?

Source: Starbucks® Corporation



11. *Investments.* A business class divided an imaginary investment of \$80,000 among three mutual funds. The first fund grew by 2%, the second by 6%, and the third by 3%. Total earnings were \$2250. The earnings from the first fund were \$150 more than the earnings from the third. How much was invested in each fund?

12. *Crying Rate.* The sum of the average number of times that a man, a woman, and a one-year-old child cry each month is 71.7. A one-year-old cries 46.4 more times than a man. The average number of times that a one-year-old cries per month is 28.3 more than the average number of times combined that a man and a woman cry. What is the average number of times per month that each cries?

13. ***Veterinary Expenditure.*** The sum of the average amounts Americans spent, per animal, for veterinary expenses for dogs, cats, and birds in a recent year was \$290. The average expenditure per dog exceeded the sum of the averages for cats and birds by \$110. The amount spent per cat was 9 times the amount spent per bird. Find the average amount spent on each type of animal.

Source: American Veterinary Medical Association

14. ***Welding Rates.*** Eldon, Dana, and Casey can weld 74 linear feet per hour when working together. Eldon and Dana together can weld 44 linear feet per hour, while Eldon and Casey can weld 50 linear feet per hour. How many linear feet per hour can each weld alone?

15. ***Nutrition.*** A dietician in a hospital prepares meals under the guidance of a physician. Suppose that for a particular patient a physician prescribes a meal to have 800 calories, 55 g of protein, and 220 mg of vitamin C. The dietician prepares a meal of roast beef, baked potato, and broccoli according to the data in the following table.

FOOD	CALORIES	PROTEIN (in grams)	VITAMIN C (in milligrams)
Roast Beef, 3 oz	300	20	0
Baked Potato	100	5	20
Broccoli, 156 g	50	5	100

How many servings of each food are needed in order to satisfy the doctor's orders?

16. ***Nutrition.*** Repeat Exercise 15 but replace the broccoli with asparagus, for which one 180-g serving contains 50 calories, 5 g of protein, and 44 mg of vitamin C. Which meal would you prefer eating?

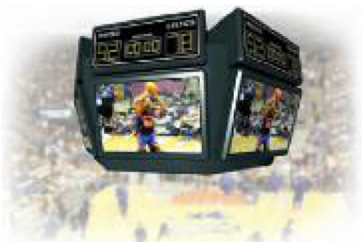
17. ***Lens Production.*** When Sight-Rite's three polishing machines, A, B, and C, are all working, 5700 lenses can be polished in one week. When only A and B are working, 3400 lenses can be polished in one week. When only B and C are working, 4200 lenses can be polished in one week. How many lenses can be polished in a week by each machine alone?

18. ***Nutrition Facts.*** A meal at Subway consisting of a 6-in. turkey breast sandwich, a bowl of minestrone soup, and a chocolate chip cookie contains 580 calories. The number of calories in the sandwich is 20 less than in the soup and the cookie together. The cookie has 120 calories more than the soup. Find the number of calories in each item.
Source: Subway

19. ***Golf.*** On an 18-hole golf course, there are par-3 holes, par-4 holes, and par-5 holes. A golfer who shoots par on every hole has a total of 70. There are twice as many par-4 holes as there are par-5 holes. How many of each type of hole are there on the golf course?

20. ***Golf.*** On an 18-hole golf course, there are par-3 holes, par-4 holes, and par-5 holes. A golfer who shoots par on every hole has a total of 72. The sum of the number of par-3 holes and the number of par-5 holes is 8. How many of each type of hole are there on the golf course?

21. ***Basketball Scoring.*** The New York Knicks once scored a total of 92 points on a combination of 2-point field goals, 3-point field goals, and 1-point foul shots. Altogether, the Knicks made 50 baskets and 19 more 2-pointers than foul shots. How many shots of each kind were made?



22. ***History.*** Find the year in which the first U.S. transcontinental railroad was completed. The following are some facts about the number. The sum of the digits in the year is 24. The ones digit is 1 more than the hundreds digit. Both the tens and the ones digits are multiples of 3.

Skill Maintenance

In each of Exercises 23–30, fill in the blank with the correct term from the given list. Some of the choices may not be used.

23. The expression $x \leq q$ means x is ______________ q. [2.8a]

24. The expression $x \geq q$ means x is ______________ q. [2.8a]

25. The graph of a(n) ______________ equation is a line. [3.2a]

26. When the slope of a line is ______________, the graph of the line slants down from left to right. [3.4a]

27. A(n) ______________ system of equations has at least one solution. [8.1a]

28. Two lines are ______________ if the product of their slopes is -1. [7.4d]

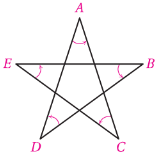
29. The ______________ of the graph of $f(x) = mx + b$ is the point $(0, b)$. [7.3a]

30. When the slope of a line is zero, the graph of the line is ______________. [7.3b]

parallel
perpendicular
consistent
inconsistent
linear
x-intercept
y-intercept
positive
zero
negative
vertical
horizontal
at least
at most

Synthesis

31. Find the sum of the angle measures at the tips of the star in this figure.



32. ***Sharing Raffle Tickets.*** Hal gives Tom as many raffle tickets as Tom has and Gary as many as Gary has. In like manner, Tom then gives Hal and Gary as many tickets as each then has. Similarly, Gary gives Hal and Tom as many tickets as each then has. If each finally has 40 tickets, with how many tickets does Tom begin?

33. ***Digits.*** Find a three-digit positive integer such that the sum of all three digits is 14, the tens digit is 2 more than the ones digit, and if the digits are reversed, the number is unchanged.

34. ***Ages.*** Tammy's age is the sum of the ages of Carmen and Dennis. Carmen's age is 2 more than the sum of the ages of Dennis and Mark. Dennis's age is four times Mark's age. The sum of all four ages is 42. How old is Tammy?

CHAPTER 8

Summary and Review

Key Terms and Formulas

system of equations, p. 570
solution of a system of equations, p. 570
consistent system of equations, p. 573
inconsistent system of equations, p. 573
dependent equations, p. 574
independent equations, p. 574
substitution method, p. 579
elimination method, p. 585
linear equation in three variables, p. 610

Motion formula: $d = rt$

Concept Reinforcement

Determine whether each statement is true or false.

______ **1.** A system of equations with infinitely many solutions is inconsistent. [8.1a]

______ **2.** It is not possible for the equations in an inconsistent system of two equations to be dependent. [8.1a]

______ **3.** When $(0, b)$ is a solution of each equation in a system of two equations, the graphs of the two equations have the same y-intercept. [8.1a]

______ **4.** The system of equations $x = 4$ and $y = -4$ is inconsistent. [8.1a]

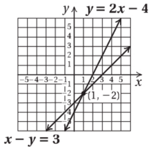
Important Concepts

Objective 8.1a Solve a system of two linear equations or two functions by graphing and determine whether a system is consistent or inconsistent and whether the equations in a system are dependent or independent.

Example Solve this system of equations graphically. Then classify the system as consistent or inconsistent and the equations as dependent or independent.

$$x - y = 3,$$
$$y = 2x - 4$$

We graph the equations.



The point of intersection appears to be $(1, -2)$. This checks in both equations, so it is the solution. The system has one solution, so it is consistent and the equations are independent.

Practice Exercise

1. Solve this system of equations graphically. Then classify the system as consistent or inconsistent and the equations as dependent or independent.

$$x + 3y = 1,$$
$$x + y = 3$$

Objective 8.2a Solve systems of equations in two variables by the substitution method.

Example Solve the system

$$x - 2y = 1, \quad \textbf{(1)}$$
$$2x - 3y = 3. \quad \textbf{(2)}$$

We solve equation (1) for x, since the coefficient of x is 1 in that equation:

$$x - 2y = 1$$
$$x = 2y + 1. \quad \textbf{(3)}$$

Next, we substitute for x in equation (2) and solve for y:

$$\begin{aligned} 2x - 3y &= 3 \\ 2(2y + 1) - 3y &= 3 \\ 4y + 2 - 3y &= 3 \\ y + 2 &= 3 \\ y &= 1. \end{aligned}$$

Then we substitute 1 for y in equation (1), (2), or (3) and find x. We choose equation (3) since it is already solved for x:

$$x = 2y + 1 = 2 \cdot 1 + 1 = 2 + 1 = 3.$$

Check:

$$\begin{array}{r|l} x - 2y & 1 \\ \hline 3 - 2 \cdot 1 & ?\ 1 \\ 3 - 2 & \\ 1 & \text{TRUE} \end{array} \qquad \begin{array}{r|l} 2x - 3y & 3 \\ \hline 2 \cdot 3 - 3 \cdot 1 & ?\ 3 \\ 6 - 3 & \\ 3 & \text{TRUE} \end{array}$$

The ordered pair (3, 1) checks in both equations, so it is the solution of the system of equations.

Practice Exercise

2. Solve the system

$$2x + y = 2,$$
$$3x + 2y = 5.$$

Objective 8.3a Solve systems of equations in two variables by the elimination method.

Example Solve the system

$$2a + 3b = -1, \quad \textbf{(1)}$$
$$3a + 2b = 6. \quad \textbf{(2)}$$

We could eliminate either a or b. In this case, we decide to eliminate the a-terms. We multiply equation (1) by 3 and equation (2) by -2 and then add and solve for b:

$$\begin{aligned} 6a + 9b &= -3 \\ -6a - 4b &= -12 \\ \hline 5b &= -15 \\ b &= -3. \end{aligned}$$

Next, we substitute -3 for b in either of the original equations:

$$\begin{aligned} 2a + 3b &= -1 \quad \textbf{(1)} \\ 2a + 3(-3) &= -1 \\ 2a - 9 &= -1 \\ 2a &= 8 \\ a &= 4. \end{aligned}$$

The ordered pair $(4, -3)$ checks in both equations, so it is a solution of the system of equations.

Practice Exercise

3. Solve the system

$$2x + 3y = 5,$$
$$3x + 4y = 6.$$

Objective 8.4a Solve applied problems involving total value and mixture using system of two equations.

Example To start a small business, Michael took two loans totaling \$18,000. One of the loans was at 7% and the other at 8%. After one year, Michael owed \$1365 in interest. What was the amount of each loan?

1. **Familiarize.** We let x and y represent the two loans. Next we organize the information in a table and use the simple interest formula, $I = Prt$.

	LOAN 1	LOAN 2	TOTAL
PRINCIPAL	x	y	\$18,000
RATE OF INTEREST	7%	8%	
TIME	1 year	1 year	
INTEREST	7%x, or $0.07x$	8%y, or $0.08y$	\$1365

2. **Translate** The total of the loans is found in the first row of the table. This gives us one equation:

$$x + y = 18{,}000.$$

From the last row of the table, we see that the interest totals \$1365. This gives us a second equation:

$$0.07x + 0.08y = 1365.$$

3. **Solve.** We solve the resulting system of equations:

$$x + y = 18{,}000, \quad \textbf{(1)}$$
$$0.07x + 0.08y = 1365. \quad \textbf{(2)}$$

We multiply by -0.07 on both sides of equation (1) and add:

$$\begin{aligned} -0.07x - 0.07y &= -1260 \\ 0.07x + 0.08y &= 1365 \quad (2) \\ \hline 0.01y &= 105 \quad \text{Adding} \\ y &= 10{,}500. \quad \text{Solving for } y \end{aligned}$$

Then

$$x + 10{,}500 = 18{,}000 \quad \text{Substituting 10,500 for } y \text{ in equation (1)}$$
$$x = 7500. \quad \text{Solving for } x$$

We find that $x = 7500$ and $y = 10{,}500$.

4. **Check.** The sum is \$7500 + \$10,500, or \$18,000. The interest from \$7500 at 7% for one year is 7%(\$7500), or \$525. The interest from \$10,500 at 8% for one year is 8%(10,500), or \$840. The total interest is \$525 + \$840, or \$1365. The numbers check in the problem.

5. **State.** Michael took loans of \$7500 at 7% and \$10,500 at 8%.

Practice Exercise

4. Jaretta made two investments totaling \$23,000. In one year, these investments yielded \$1237 in simple interest. Part of the money was invested at 6% and the rest at 5%. How much was invested at each rate?

Objective 8.5a Solve systems of three equations in three variables.

Example Solve:

$$\begin{aligned} x - y - z &= -2, && \textbf{(1)} \\ 2x + 3y + z &= 3, && \textbf{(2)} \\ 5x - 2y - 2z &= -1. && \textbf{(3)} \end{aligned}$$

The equations are in standard form and do not contain decimals or fractions. We choose to eliminate z since the z-terms in equations (1) and (2) are opposites. First, we add these two equations:

$$\begin{array}{l} x - y - z = -2 \\ \underline{2x + 3y + z = 3} \\ 3x + 2y \qquad = 1. \quad \textbf{(4)} \end{array}$$

Next, we multiply equation (2) by 2 and add it to equation (3) to eliminate z from another pair of equations:

$$\begin{array}{l} 4x + 6y + 2z = 6 \\ \underline{5x - 2y - 2z = -1} \\ 9x + 4y \qquad = 5. \quad \textbf{(5)} \end{array}$$

Now we solve the system consisting of equations (4) and (5). We multiply equation (4) by -2 and add:

$$\begin{array}{r} -6x - 4y = -2 \\ \underline{9x + 4y = 5} \\ 3x \qquad = 3 \\ x = 1. \end{array}$$

Then we use either equation (4) or (5) to find y:

$$\begin{aligned} 3x + 2y &= 1 && \textbf{(4)} \\ 3 \cdot 1 + 2y &= 1 \\ 3 + 2y &= 1 \\ 2y &= -2 \\ y &= -1. \end{aligned}$$

Finally, we use one of the original equations to find z:

$$\begin{aligned} 2x + 3y + z &= 3 && \textbf{(2)} \\ 2 \cdot 1 + 3(-1) + z &= 3 \\ -1 + z &= 3 \\ z &= 4. \end{aligned}$$

Check:

$$\begin{array}{r|l} x - y - z & -2 \\ \hline 1 - (-1) - 4 & ?\ -2 \\ 1 + 1 - 4 & \\ -2 & \quad \text{TRUE} \end{array} \qquad \begin{array}{r|l} 2x + 3y + z & 3 \\ \hline 2 \cdot 1 + 3(-1) + 4 & ?\ 3 \\ 2 - 3 + 4 & \\ 3 & \quad \text{TRUE} \end{array}$$

$$\begin{array}{r|l} 5x - 2y - 2z & -1 \\ \hline 5 \cdot 1 - 2(-1) - 2 \cdot 4 & ?\ -1 \\ 5 + 2 - 8 & \\ -1 & \quad \text{TRUE} \end{array}$$

The ordered triple $(1, -1, 4)$ checks in all three equations, so it is the solution of the system of equations.

Practice Exercise

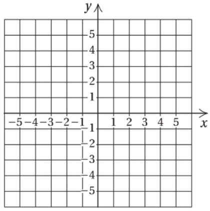
5. Solve:

$$\begin{aligned} x - y + z &= 9, \\ 2x + y + 2z &= 3, \\ 4x + 2y - 3z &= -1. \end{aligned}$$

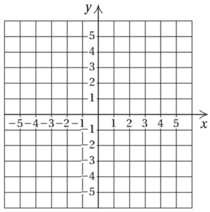
Review Exercises

Solve graphically. Then classify the system as consistent or inconsistent and the equations as dependent or independent. [8.1a]

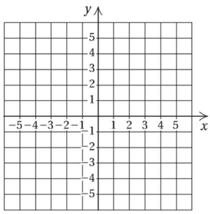
1. $4x - y = -9,$
$x - y = -3$



2. $15x + 10y = -20,$
$3x + 2y = -4$



3. $y - 2x = 4,$
$y - 2x = 5$



Solve by the substitution method. [8.2a]

4. $2x - 3y = 5,$
$x = 4y + 5$

5. $y = x + 2,$
$y - x = 8$

6. $7x - 4y = 6,$
$y - 3x = -2$

Solve by the elimination method. [8.3a]

7. $x + 3y = -3,$
$2x - 3y = 21$

8. $3x - 5y = -4,$
$5x - 3y = 4$

9. $\frac{1}{3}x + \frac{2}{9}y = 1,$
$\frac{3}{2}x + \frac{1}{2}y = 6$

10. $1.5x - 3 = -2y,$
$3x + 4y = 6$

11. ***Air Travel.*** An airplane flew for 3 hr with a 30-mph tailwind. The return flight against the same wind took 4.5 hr. Find the speed of the plane in still air. [8.4b]

12. ***Spending Choices.*** Sean has \$86 to spend. He can spend all of it on one CD and two DVDs, or he can buy two CDs and one DVD and have \$16 left over. What is the price of a CD? of a DVD? [8.4a]

13. ***Orange Drink Mixtures.*** "Orange Thirst" is 15% orange juice and "Quencho" is 5% orange juice. How many liters of each should be combined in order to get 10 L of a mixture that is 10% orange juice? [8.4a]

14. ***Train Travel.*** A train leaves Watsonville at noon traveling north at 44 mph. One hour later, another train, going 52 mph, travels north on a parallel track. How many hours will the second train travel before it overtakes the first train? [8.4b]

Solve. [8.5a]

15. $$\begin{aligned} x + 2y + z &= 10, \\ 2x - y + z &= 8, \\ 3x + y + 4z &= 2 \end{aligned}$$

16. $$\begin{aligned} 3x + 2y + z &= 1, \\ 2x - y - 3z &= 1, \\ -x + 3y + 2z &= 6 \end{aligned}$$

17. $$\begin{aligned} 2x - 5y - 2z &= -4, \\ 7x + 2y - 5z &= -6, \\ -2x + 3y + 2z &= 4 \end{aligned}$$

18. $$\begin{aligned} x + y + 2z &= 1, \\ x - y + z &= 1, \\ x + 2y + z &= 2 \end{aligned}$$

19. ***Triangle Measure.*** In triangle ABC, the measure of angle A is four times the measure of angle C, and the measure of angle B is 45° more than the measure of angle C. What are the measures of the angles of the triangle? [8.6a]

20. ***Money Mixtures.*** Elaine has \$194, consisting of \$20, \$5, and \$1 bills. The number of \$1 bills is 1 less than the total number of \$20 and \$5 bills. If she has 39 bills in her purse, how many of each denomination does she have? [8.6a]

21. Solve using the elimination method:

$$\begin{aligned} x - y &= -9, \\ y - 2x &= 9. \end{aligned}$$

The first coordinate of the solution is which of the following? [8.3a]

A. 9 **B.** -9
C. 0 **D.** $\frac{9}{2}$

22. The sum of two numbers is -2. The sum of twice one number and the other is 4. One number is which of the following? [8.3b]

A. -6 **B.** 2
C. 6 **D.** 8

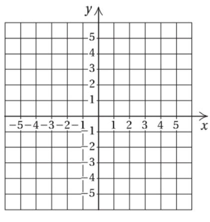
23. ***Distance Traveled.*** Two cars leave Martinsville traveling in opposite directions. One car travels at a speed of 50 mph and the other at 60 mph. In how many hours will they be 275 mi apart? [8.4b]

A. 2.5 hr **B.** 3 hr
C. 3.5 hr **D.** 4 hr

Synthesis

24. Solve graphically: [7.1c], [8.1a]

$$\begin{aligned} y &= x + 2, \\ y &= x^2 + 2. \end{aligned}$$



Understanding Through Discussion and Writing

1. Write a problem for a classmate to solve. Design the problem so the answer is "The florist sold 14 hanging baskets and 9 flats of petunias." [8.4a]

2. Exercise 14 in Exercise Set 8.6 can be solved mentally after a careful reading of the problem. Explain how this can be done. [8.6a]

3. ***Ticket Revenue.*** A pops-concert audience of 100 people consists of adults, senior citizens, and children. The ticket prices are \$10 each for adults, \$3 each for senior citizens, and \$0.50 each for children. The total amount of money taken in is \$100. How many adults, senior citizens, and children are in attendance? Does there seem to be some information missing? Do some careful reasoning and explain. [8.6a]

CHAPTER

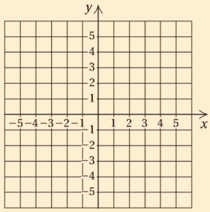
8 Test

For Extra Help

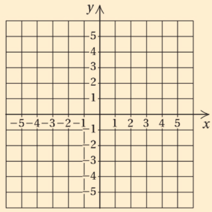
Step-by-step test solutions are found on the Chapter Test Prep Videos available via the Video Resources on DVD, in MyMathLab, and on YouTube (search "BittingerIntroInter" and click on "Channels").

Solve graphically. Then classify the system as consistent or inconsistent and the equations as dependent or independent.

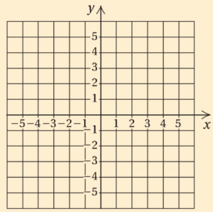
1. $y = 3x + 7,$
$3x + 2y = -4$



2. $y = 3x + 4,$
$y = 3x - 2$



3. $y - 3x = 6,$
$6x - 2y = -12$



Solve by the substitution method.

4. $4x + 3y = -1,$
$y = 2x - 7$

5. $x = 3y + 2,$
$2x - 6y = 4$

6. $x + 2y = 6,$
$2x + 3y = 7$

7. $t = 2 - r,$
$3r - 2t = 36$

Solve by the elimination method.

8. $2x + 5y = 3,$
$-2x + 3y = 5$

9. $x + y = -2,$
$4x - 6y = -3$

10. $\frac{2}{3}x - \frac{4}{5}y = 1,$
$\frac{1}{3}x - \frac{2}{5}y = 2$

11. $0.3a - 0.4b = 11,$
$0.7a + 1.2b = -17$

Solve.

12. *Tennis Court.* The perimeter of a standard tennis court used for playing doubles is 288 ft. The width of the court is 42 ft less than the length. Find the length and the width.

13. *Air Travel.* An airplane flew for 5 hr with a 20-km/h tailwind and returned in 7 hr against the same wind. Find the speed of the plane in still air.

14. *Chicken Dinners.* High Flyin' Wings charges $12 for a bucket of chicken wings and $7 for a chicken dinner. After filling 28 orders for buckets and dinners during a football game, the waiters had collected $281. How many buckets and how many dinners did they sell?

15. *Mixing Solutions.* A chemist has one solution that is 20% salt and a second solution that is 45% salt. How many liters of each should be used in order to get 20 L of a solution that is 30% salt?

16. Solve:

$$\begin{aligned} 6x + 2y - 4z &= 15, \\ -3x - 4y + 2z &= -6, \\ 4x - 6y + 3z &= 8. \end{aligned}$$

17. *Repair Rates.* An electrician, a carpenter, and a plumber are hired to work on a house. The electrician earns $21 per hour, the carpenter $19.50 per hour, and the plumber $24 per hour. The first day on the job, they worked a total of 21.5 hr and earned a total of $469.50. If the plumber worked 2 hr more than the carpenter did, how many hours did the electrician work?

18. A business class divided an imaginary $30,000 investment among three funds. The first fund grew 2%, the second grew 3%, and the third grew 5%. Total earnings were $990. The earnings from the third fund were $280 more than the earnings from the first. How much was invested at 5%?

A. $9000 **B.** $10,000 **C.** $11,000 **D.** $12,000

Synthesis

19. The graph of the function $f(x) = mx + b$ contains the points $(-1, 3)$ and $(-2, -4)$. Find m and b.

CHAPTERS

1–8

Cumulative Review

Perform the indicated operations and simplify.

1. $(3x^4 - 2y^5)(3x^4 + 2y^5)$

2. $(x^2 + 4)^2$

3. $\left(2x + \frac{1}{4}\right)\left(4x - \frac{1}{2}\right)$

4. $\frac{x}{2x - 1} - \frac{3x + 2}{1 - 2x}$

5. $(3x^2 - 2x^3) - (x^3 - 2x^2 + 5) + (3x^2 - 5x + 5)$

6. $\frac{2x + 2}{3x - 9} \cdot \frac{x^2 - 8x + 15}{x^2 - 1}$

7. $\frac{2x^2 - 2}{2x^2 + 7x + 3} \div \frac{4x - 4}{2x^2 - 5x - 3}$

8. $(3x^3 - 2x^2 + x - 5) \div (x - 2)$

Factor completely.

9. $3 - 12x^8$

10. $12t - 4t^2 - 48t^4$

11. $6x^2 - 28x + 16$

12. $4x^3 + 4x^2 - x - 1$

13. $16x^4 - 56x^2 + 49$

14. $x^2 + 3x - 180$

15. Find the slope and the y-intercept of $5y - 4x = 20$.

16. Find an equation of the line with slope -3 and containing the point $(5, 2)$.

17. Find an equation of the line parallel to $3x - 9y = 2$ and containing the point $(-6, 2)$.

18. Determine whether the graphs of the given lines are parallel, perpendicular, or neither.

$$x - 2y = 4,$$
$$4x + 2y = 1$$

Solve.

19. $x^2 = -17x$

20. $\frac{1}{4}x + \frac{2}{3}x = \frac{2}{3} - \frac{3}{4}x$

21. $\frac{1}{x} + \frac{2}{3} = \frac{1}{4}$

22. $x^2 - 30 = x$

23. $-4(x + 5) \geq 2(x + 5) - 3$

24. $\frac{x}{x - 1} - \frac{x}{x + 1} = \frac{1}{2x - 2}$

25. Solve $4A = pr + pq$ for p.

Solve.

26. $3x + 4y = 4,$
$x = 2y + 2$

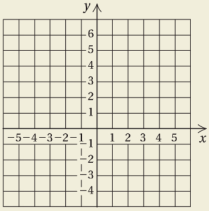
27. $3x + y = 2,$
$6x - y = 7$

28. $4x + 3y = 5,$
$3x + 2y = 3$

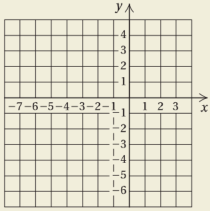
29. $x - y + z = 1,$
$2x + y + z = 3,$
$x + y - 2z = 4$

Graph on a plane.

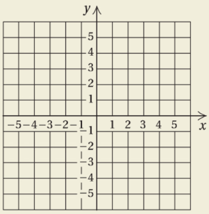
30. $3y = 9$



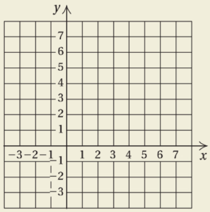
31. $f(x) = -\frac{1}{2}x - 3$



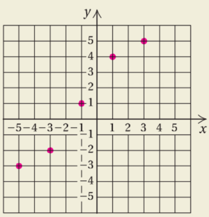
32. $3x - 1 = y$



33. $3x + 5y = 15$



34. For the function f whose graph is shown below, determine **(a)** the domain, **(b)** the range, **(c)** $f(-3)$, and **(d)** any input for which $f(x) = 5$.



35. Find the domain of the function given by

$$f(x) = \frac{7}{2x - 1}.$$

36. Given $g(x) = 1 - 2x^2$, find $g(-1)$, $g(0)$, and $g(3)$.

37. *Mixing Solutions.* A technician wants to mix one solution that is 15% alcohol with another solution that is 25% alcohol in order to get 30 L of a solution that is 18% alcohol. How much of each solution should be used?

38. *Utility Cost.* One month Ladi and Bo spent \$680 for electricity, rent, and telephone. The electric bill was one-fourth of the rent and the rent was \$400 more than the phone bill. How much was the electric bill?

39. *Quality Control.* A sample of 150 resistors contained 12 defective resistors. How many defective resistors would you expect to find in a sample of 250 resistors?

40. *Rectangle Dimensions.* The length of a rectangle is 3 m greater than the width. The area of the rectangle is 180 m^2. Find the length and the width.

41. *Apparent Size.* The apparent size A of an object varies inversely as the distance d of the object from the eye. You are sitting at a concert 100 ft from the stage. The musicians appear to be 4 ft tall. How tall would they appear to be if you were sitting 1000 ft away in the lawn seats?

42. *Angles of a Triangle.* The second angle of a triangle is twice as large as the first. The third angle is 48° less than the sum of the other two angles. Find the measures of the angles.

Synthesis

43. *Radio Advertising.* An automotive dealer discovers that when \$1000 is spent on radio advertising, weekly sales increase by \$101,000. When \$1250 is spent on radio advertising, weekly sales increase by \$126,000. Assuming that sales increase according to a linear function, by what amount would sales increase when \$1500 is spent on radio advertising?

44. Given that $f(x) = mx + b$ and that $f(5) = -3$ when $f(-4) = 2$, find m and b.